On an error in defining temperature feedback

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Abstract

CLIMATOLOGY, in borrowing feedback method from control theory, misapplies a concept from experimental science in an observational-science setting by defining temperature feedback as responding only to perturbations of reference temperature. One implication of this error is that, impossibly, the feedback fraction due to warming from noncondensing greenhouse gases exceeds that due to emission temperature by 1-2 orders of magnitude. Another implication is that feedback response constitutes up to 90% of midrange Charney sensitivity (equilibrium sensitivity to doubled CO_2 after feedback has acted) and of the uncertainty therein. In reality, feedback responds to the entire reference signal, which, in climate, is the sum of emission temperature and all natural as well as anthropogenic reference sensitivities. The system-gain factor, the ratio of equilibrium to reference temperatures and not (as now) of perturbations only, is insensitive even to large uncertainties in those temperatures. It was 1.1 in 1850 and in 2011. Revised Charney sensitivity, the product of the 1.05 K reference sensitivity to doubled CO_2 and the system-gain factor 1.1, falls on 1.15 [1.10, 1.25] K, confirmed empirically from ten published estimates of anthropogenic forcing. Monte Carlo simulations compared the CMIP5 and revised confidence intervals. An author and a government laboratory verified the theory with test apparatus. Current Charney-sensitivity projections on 3.35 [2.1, 4.7] K are excessive. Even without mitigation, global warming sufficient to cause net harm is unlikely.

1. Introduction

Global warming is not occurring as rapidly as predicted. The Intergovernmental Panel on Climate Change (IPCC 1990, pp. *xii*, *xiv*) had projected $\sim 0.3 \pm 0.1$ K decade^{-1} medium-term warming, but, after only 0.17 K decade^{-1} was observed from 1990-2011 (HadCRUT4: Morice et al. 2012), for the first time IPCC (2013) replaced outputs from general-circulation models (GCMs) with its "expert judgment", near-halving its medium-term projection to 0.17 ± 0.1 K decade^{-1}. However, IPCC did not reduce its [1.5, 4.5] K 95%-confidence interval of Charney sensitivity, the standard metric, which is equilibrium sensitivity to radiative forcing from doubled CO_2 after all temperature feedbacks of sub-decadal duration have operated. That interval remains as in Charney et al. (1979, p. 4), the first modern sensitivity study. The 3.35 [2.1, 4.7] K interval of projected Charney sensitivities in the fifth-generation models of the Climate Model Intercomparison Project (CMIP5: Andrews et al. 2012) is also excessive when compared with observed warming since 1850 (Fig. 1). Accordingly, the question whether there subsists a systemic error leading to much-overstated projections of global warming was investigated.

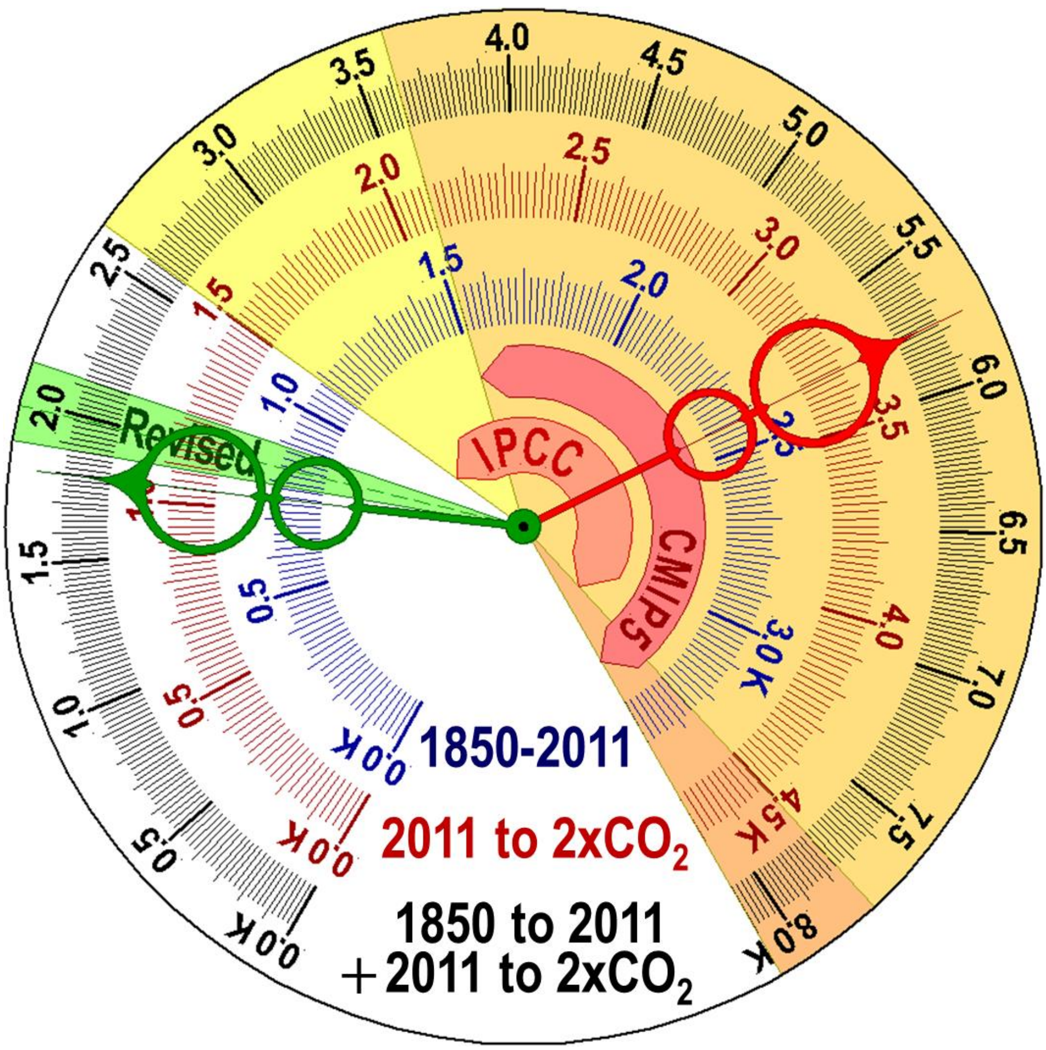

FIG. 1. Overlapping projections by IPCC (2013) and CMIP5 (Andrews et al. 2012) of global warming from 1850-2011 **(blue scale)**, in response to doubled CO_2 **(red scale)** and the sum of these two (**black scale**) greatly exceed warming equivalent to the 0.75 K observed from 1850-2011 (HadCRUT4: **green needle**). The 3.35 K CMIP5 mid-range Charney sensitivity **(red needle)** implies 2.4 K anthropogenic warming by 2011, about thrice observation. The revised warming interval derived herein **(pale green region)** is consistent with observed warming to 2011 (**green needle**).

The small uncertainty of ±10% (IPCC 2013, p 676, §8.3.2.1; *cf*. Cess et al. 1993) in reference sensitivity (sensitivity before allowing for temperature feedback) and the large uncertainty in the feedback response indicate that, if there be some such error, it may lie in the treatment of temperature feedback. IPCC (2013) indicates the significance of feedback by mentioning it more than 1000 times. Midrange feedback response is currently thought to account for some 85% (Vial et al. 2013) of the 3 K uncertainty in equilibrium sensitivity (sensitivity after feedback has acted), while feedback is currently thought to contribute 70% [50%, 90%] of equilibrium sensitivity. Uncertainty in feedback strength arising from an error in the definition of temperature feedback will be shown to be the chief reason why the published intervals of projected Charney sensitivity (IPCC 2013) remain broad and have resisted constraint for 40 years, leading to a threefold overstatement of projected midrange global warming.

2. Erroneous definition of temperature feedback

Feedback in dynamical systems is correctly defined (Black 1934, Bode 1945, Roe 2009) as responding to the entire reference signal, which is the sum of the input signal (in climate, emission temperature) and any perturbations thereof (reference sensitivities to natural as well as anthropogenic perturbations). This long-established definition had its origin in electronic network analysis, from which sprang the branch of engineering physics known as control theory. For a century, it has been so often applied to dynamical systems throughout the sciences, from telephone circuits to rocketry, as to be unimpugnable.

Feedback theory has long been incorporated into climatology by explicit reference to its origin in electronics (e.g. Hansen et al. 1984; Schlesinger 1985, IPCC 1990 p. *xiv*, Roe 2009, Schmidt et al. 2010, Monckton of Brenchley 2015b). However, climatology erroneously defines feedback as responding to perturbations only; the definition does not encompass feedback response to emission temperature.

Feedback is at once a consequence and an instrument of causality. In physics, natural laws describe the evolution and interaction of quantities – measurable properties of the universe at a particular point in space-time. Causality describes the direction of their interaction and evolution. If a system is sufficiently complex, causal interactions may take the form of chains and even loops, such as the feedback loop. Little complexity is required as a condition for the establishment of a causal loop. For instance, a solid at 0 K upon which a constant source of radiation is incident will at first absorb more energy than it emits. The difference will warm the solid, diminishing the radiative imbalance and slowing the rate of warming. Eventually, radiative equilibrium will be attained and the solid will hold its new temperature. Bony et al. (2006), describing this feedback process as "the most important feedback in the climate system", imply that at least one feedback will operate at any given temperature.

Feedbacks are an emergent phenomenon. Therefore, they may not be arbitrarily defined. Since causality is a universal property, the feedback principles underlying that causality are universal properties applicable to all dynamical systems, from electronic circuits to climate.

Where an approximately constant radiative input acts upon a dynamical system, three outcomes are possible. First, where the output is constant and exceeds the input, any feedback processes that are present will have responded to the input by amplifying the output. Secondly, where the output is constant and less than the input, the feedback processes will have responded to the input by attenuating the output. Thirdly, where the system is unstable, the output will never have attained a constant value.

In the Earth's climate, the observed global mean surface temperature at any time exceeds the reference temperature that would hypothetically obtain without feedback. Accordingly, net-positive temperature feedback processes subsist, amplifying the reference temperature. At any given time t at which the climate is in radiative equilibrium (or at which due adjustment for any radiative disequilibrium at time t establishes a theoretical equilibrium), **reference temperature R_t** is defined as the absolute temperature that would obtain in the absence of feedback. **Equilibrium temperature E_t** is defined as the absolute temperature that obtains once the climate has returned to equilibrium after short-acting feedbacks have acted upon R_t. Thus, E_t is a function $E(R)$ of R_t.

Ex definitione, the **absolute system-gain factor A_t**, which is the function $A(E, R)$ of E_t, R_t that encompasses the entire operation of temperature feedback at any time t of radiative equilibrium, is the ratio of E_t to R_t (Eq. 1); and A_t thus derived applies regardless of the shape of the function $E(R)$.

$$A_t \coloneqq E_t / R_t \quad | \quad \text{Absolute system-gain factor} \qquad (1)$$
$$a_t \coloneqq \Delta E_{t-1} / \Delta R_{t-1} \quad | \quad \text{Partial system-gain factor} \qquad (2)$$
$$A_t \leq a_t \quad | \quad \text{At radiative equilibrium} \qquad (3)$$

Climatology has hitherto eschewed the absolute system-gain factor A_t, preferring to rely exclusively upon the **partial system-gain factor a_t**, the function $a(\Delta E, \Delta R)$ of equilibrium sensitivity ΔE_{t-1}, which is the ratio of **equilibrium sensitivity ΔE_{t-1}** to **reference sensitivity ΔR_{t-1}** (Eq. 2).

Originally, GCMs' outputs served as the inputs to a separate, simple model incorporating Eq. (2), from which equilibrium sensitivities were derived. Though GCMs now derive equilibrium sensitivities without reference to Eq. (2), it has hitherto escaped attention that such feedback processes as subsist in the climate at time t must perforce act upon the entire reference temperature R_t, and not merely upon some arbitrarily-chosen perturbation ΔR_{t-1}. Since the absolute temperatures E_t, R_t whose ratio is A_t exceed by two orders of magnitude the sensitivities $\Delta E_{t-1}, \Delta E_{t-1}$ whose ratio is a_t, even large uncertainties in E_t, R_t entail only a small uncertainty in A_t. By contrast, even small uncertainties in $\Delta R_{t-1}, \Delta E_{t-1}$ entail a large uncertainty in a_t. This is is the chief reason why constraint of equilibrium sensitivity has hitherto proven elusive. Eq. (1) permits a considerably tighter and more reliable constraint of equilibrium sensitivities ΔE_{t-1} than Eq. (2).

Though differentiation tends to increase signal noise owing to high-pass behavior, the uncertainty introduced by this consideration is small because, as will be shown, if the shape of the function $E(R)$ is exponential the exponent x barely exceeds unity, so that the equilibrium-response function $E(R)$ is near-linear. For this reason, at any time t when the climate is in equilibrium, $A_t < a_t$ where $E(R)$ is a growth function, while $A_t = a_t$ where $E(R)$ is linear (Eq. 3).

Eqs. (4-6) demonstrate the relationship between the two system-gain factors A, a_t.

$$E_t = R_t \, A_t \quad | \quad \text{At equilibrium} \qquad (4)$$
$$E_{t-1} = R_{t-1} \, A_{t-1} \quad | \quad \text{At previous equilibrium} \qquad (5)$$

$$\Delta E_{t-1} = E_t - E_{t-1} = R_t A_t - R_{t-1} A_{t-1} \quad | \quad \text{Eq. (4) – Eq. (5):} \qquad (6)$$
$$= (R_t - R_{t-1}) \, a_t = \Delta R_{t-1} \, a_t \quad | \quad \text{partial system-gain equation}$$

From Eq. (2) and empirical data, a_t may be estimated via Eq. (7). However, even though E_t, R_t in 1850 are well constrained, there is considerable uncertainty as to their values thereafter. Therefore, Eq. (7) has not proven useful hitherto, and the broad interval of projected equilibrium sensitivities has resisted constraint.

$$a_t = \frac{E_t - E_{t-1}}{R_t - R_{t-1}} = \frac{\Delta E_{t-1}}{\Delta R_{t-1}} \qquad | \text{ Eq. (4) – Eq. (5)} \qquad (7)$$

GCMs derive a_t independently of Eqs. (1, 2). They simulate physical processes that give rise to temperature feedbacks, treating them as an emergent property diagnosed from models' outputs (Soden & Held 2006; Vial et al. 2013). They attempt to derive the partial system-gain factor a_t bottom-up by estimating the individual climate-relevant temperature feedbacks, treating a_t (Eq. 2) as the ratio solely of sensitivities $\Delta E_{t-1}/\Delta R_{t-1}$ but not also as the ratio (Eq. 1) of absolute temperatures E_t/R_t.

Eqs. (4-6) may be represented by a leading-order Taylor-series expansion, Eq. (10), derived via Eqs. (8, 9) (Bony et al. 2006, Roe 2009).

$$E_t - E_{t-1} = (R_t - R_{t-1})\, a_t \qquad | \text{ Rearrange Eq. (7)} \qquad (8)$$

$$E_t = E_{t-1} + (R_t - R_{t-1})\, a_t \qquad | \text{ Transpose } E_{t-1} \qquad (9)$$

$$E_t(R) = E_{t-1}(R) + (R_t - R_{t-1})\, dE/dR \qquad | \text{ Taylor-series expansion} \qquad (10)$$

Either Eq. (1) or Eq. (2) may be used diagnostically to derive ΔE_t from specified values of $\Delta R_t, A_t$, provided that A_t has first been correctly derived. However, derivation of Eq. (2) from the energy-balance equation via a Taylor-series expansion reveals nothing of the magnitude of the system-gain factor. That factor is reliably constrainable only via Eq. (1).

The definition of "climate feedback" in IPCC (2013, p. 1450) does not state that feedback processes respond to absolute reference temperature. Instead, *perturb* or *perturbation* is mentioned five times. Climatology's definition is consistent with Eq. (2) but is so restrictive as to be inconsistent with Eq. (1).

> "**Climate feedback:** An interaction in which a *perturbation* in one climate quantity causes a change in a second, and the change in the second quantity ultimately leads to an additional change in the first. A negative feedback is one in which the initial *perturbation* is weakened by the changes it causes; a positive feedback is one in which the initial *perturbation* is enhanced. In this Assessment Report, a somewhat narrower definition is often used in which the climate quantity that is *perturbed* is the global mean surface temperature, which in turn causes changes in the global radiation budget. In either case, the initial *perturbation* can either be externally forced or arise as part of internal variability." [Authors' emphases]

The error here is that the difference between a pre-existing reference temperature and a perturbation thereof is an artefact. Physics does not pass judgment on different states of the same quantity: they are as they are. To call one state a quantity and another state a perturbation is a value-judgment by the observer. If one were to define the initial reference temperature as 0 K, then every temperature > 0 K would be a perturbation of that reference temperature. Accordingly, climatology's definition is not of universal application, since the results of relying upon it depend on an arbitrary value defined as "quantity". Climatology's definition is erroneous: it fails to take advantage of the absolute system-gain factor A_t, derived in Eq. (1), which reliably constrains equilibrium sensitivities.

From GCMs' outputs, n individual temperature feedbacks $(\lambda_i)_t$ in W m^{-2} K^{-1}, summing to $\lambda_t = \sum_{i=1}^{n}(\lambda_i)_t$, are diagnosed. Where reference sensitivity ΔR_{t-1} is the product of a radiative forcing ΔQ_{t-1} and the Planck sensitivity parameter P_t (Eq. 15), Eq. (11) gives equilibrium sensitivity ΔE_{t-1}.

$$\Delta E_{t-1} = [\Delta Q_{t-1} + \lambda_t \Delta E_{t-1}]P_t = \frac{\Delta Q_{t-1} P_t}{1 - \lambda_t P_t} = \frac{\Delta R_{t-1}}{1 - f_t} = \Delta R_{t-1} a_t. \tag{11}$$

IPCC (2007, p. 631 fn.) describes Eq. (11) thus [after adjusting notation to conform hereto]:

> "… the amplification [a_t] of the global warming from a feedback sum [λ_t] (in W m^{-2} K^{-1}) with no other feedbacks operating is [$1/(1 - \lambda_t P_t$], where [P_t] is [~0.3, the reciprocal of] the 'uniform temperature' radiative cooling response (of value approximately 3.2 K W^{-1} m^2: Bony et al. 2006). If n independent feedbacks operate, [λ_t] is replaced by [$(\lambda_1)_t + (\lambda_2)_t + \cdots + (\lambda_n)_t$]."

As a direct result of climatology's error in defining temperature feedback, it has hitherto been implicitly assumed that feedbacks do not respond to reference temperature R_1 as it stood in 1850: see e.g. Hansen et al. (1981), Schlesinger (1985), IPCC (1990, p. *xiv*), Roe (2009), Schmidt et al. (2010). Attempts at bottom-up diagnosis using the partial system-gain factor a_t have led to large overstatements of the feedback fraction f_t, of the feedback response b_t $(= f_t E_t = E_t - R_t)$, of the system-gain factors A_t $(= E_t/R_t), a_t$ $(= \Delta E_{t-1}/\Delta R_{t-1})$ and thus of all equilibrium sensitivities ΔE_{t-1}.

The consequences of the error of definition are severe. It is currently thought that, owing to feedback, equilibrium sensitivity ΔE_{t-1} exceeds reference sensitivity ΔR_{t-1} by up to fourfold; in some sources, up to tenfold (e.g. Armour 2017; Friedrich et al. 2016; Johansson et al. 2015; Murphy et al. 2009; Forest et al. 2006; Andronova & Schlesinger 2001).

3. Definition of temperature feedback and related terms

Formal definitions of temperature feedback and related terms and a demonstration of the form of the system-gain factor A_t are essential. From the revised definitions, a corrected interval of equilibrium sensitivities will be derived more simply and with less uncertainty than from the defective variant a_t, and without the need to resort to GCMs (Monckton of Brenchley et al. 2015a). Terminology herein, though close to what is standard in control theory, may differ from the current climatological usage.

Feedback in any dynamical system (Fig. 2) responds to a ***reference signal*** $R_t > 0$ and modifies the ***output signal*** E_t. **Positive feedback** amplifies the output signal: **negative feedback** attenuates it. Reference and equilibrium temperatures R, E are time-dependent scalars. Since each may possess a unique value at a given time t, their subscript t indicates time-dependence.

It is generally assumed that feedbacks are time-invariant: the same input will yield similar outputs when applied at different points in time. However, where $E(R)$ is a nonlinear function the system-gain factor A may vary with different reference signals. Then A is a function of R, which is itself a function of time. Accordingly, the notation A_t is a shortened form of the notation $(A_R)_t$ for the system-gain factor corresponding to the specific value R_t of the reference temperature R at time t.

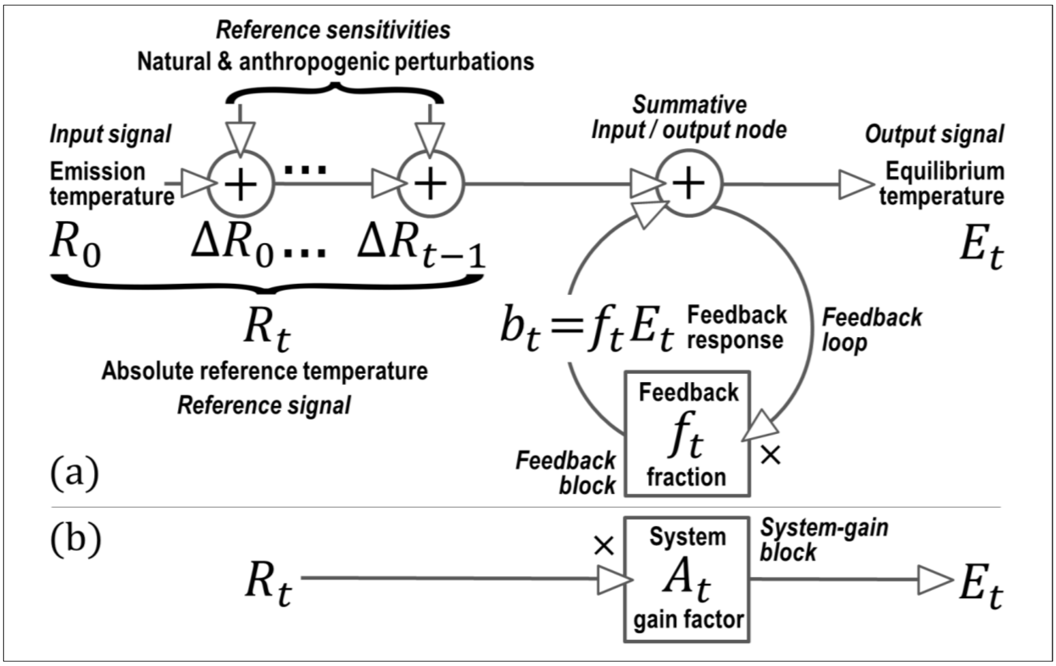

FIG. 2. The ***feedback loop*** (a) simplifies to (b), the schematic for the **system-gain factor** A_t at time t. The ***reference signal*** **(reference temperature** R_t), the sum of the ***input signal*** **(emission temperature** R_0), and all ***perturbations*** (**reference sensitivities** $\Delta R_0, \ldots \Delta R_{t-1}$), is input via the ***summative input/output node*** to the feedback loop. The ***output signal*** **(equilibrium temperature** E_t), is the sum of R_t and the **feedback response** $b_t \coloneqq f_t E_t$ ($\coloneqq E_t - R_t$). Then A_t ($= E_t / R_t$) is equal to the sum $\sum_{i=0}^{\infty} f_t^i = (1 - f_t)^{-1}$ of the infinite convergent geometric series $\{{f_t}^0 + {f_t}^1 + \cdots + {f_t}^\infty\}$ under the convergence criterion $|\, f_t \,| < 1$. The ***feedback block*** (a) and the ***system-gain block*** (b) must perforce act not only on the anthropogenic perturbation ΔR_{t-1} but on the entire reference signal R_t.

In climate, at time t, n **temperature feedback processes** $(\lambda_1)_t, (\lambda_2)_t, \ldots (\lambda_n)_t$, in Watts per square meter per Kelvin of the **reference temperature** R_t, sum to the **feedback sum** λ_t (Eq. 12).

$$\lambda_t = \sum_{i=1}^{n} (\lambda_i)_t. \tag{12}$$

The **feedback fraction** f_t of equilibrium sensitivity represented by the feedback response is the dimensionless product (Eq. 13) of λ_t and the Planck sensitivity parameter P_t, the latter in K W^{-1} m^2.

$$f_t = \lambda_t P_t. \tag{13}$$

Mean emission flux density Q_2 for total solar irradiance $S_0 = 1363.5 \text{ W m}^{-2}$ (Dewitte & Nevens 2016, cf. Mekaoui et al. 2010) and mean planetary albedo $\alpha_2 = 0.3$ (Loeb 2009) is given by Eq. (14).

$$Q_2 = S_0(1 - \alpha_2)/4 = 238.6 \text{ W m}^{-2}. \tag{14}$$

The **Planck sensitivity parameter** P_2 is, to first approximation, the first derivative of the Stefan-Boltzmann equation. It may be taken (Schlesinger 1985) as the ratio (Eq. 15) of global mean surface temperature T_2 ($= 288.4$ K today: Morice et al. 2012) to four times the mean radiative flux density Q_2 in Eq. (14). In today's climate, $P_2 \approx 0.30 \text{ K W}^{-1} \text{ m}^2$ (based on Schlesinger 1985) or $0.31 \text{ K W}^{-1} \text{ m}^2$ (Soden & Held 2006; IPCC 2007, p. 631 fn.).

$$P_2 = T_2/4Q_2 = 288.4/(4 \text{ x } 238.6) \approx 0.30 \text{ K W}^{-1} \text{ m}^2. \tag{15}$$

The reference signal before accounting for any temperature feedback is absolute global mean surface **reference temperature** R_t, the sum (in Eq. 16) of **emission temperature** R_0 in the absence of any NCGHGs and successive **reference sensitivities** $\Delta R_0, \Delta R_1, \ldots, \Delta R_{t-1}$. In Fig. 2, the feedback block visibly acts not only upon one or more of the perturbations $\Delta R_0, \ldots \Delta R_{t-1}$ of R_0, as is currently thought, but upon the entire reference signal R_t.

$$R_t \coloneqq R_{t-1} + \Delta R_{t-1} \coloneqq R_0 + \sum_{i=0}^{t-1} \Delta R_i. \tag{16}$$

Eq. (17) defines the **feedback response** b_t as the difference between **equilibrium temperature** E_t and **reference temperature** R_t, while Eq. (18) defines the **feedback fraction** f_t.

$$b_t \coloneqq f_t E_t \coloneqq E_t - R_t; \tag{17}$$

$$f_t = b_t / E_t = 1 - R_t / E_t. \tag{18}$$

Upon re-equilibration of the climate after accounting for the operation of the short-acting sensitivity-altering feedbacks (the term used in Bates 2016) represented by f_t, where **equilibrium sensitivity** is ΔE_{t-1} the output signal (Eq. 19) is absolute global mean surface **equilibrium temperature** E_t. *Ex definitione*, the **absolute system-gain factor** A_t that encompasses the entire action of feedback at time t is the ratio (in Eq. 20) of equilibrium to reference temperatures.

$$E_t = R_t + b_t \coloneqq E_{t-1} + \Delta E_{t-1} \tag{19}$$

$$A_t = E_t / R_t = (R_t + f_t E_t) / R_t = (R_t + b_t) / R_t = (1 - f_t)^{-1}. \tag{20}$$

Feedback processes or their magnitudes may vary over time. Such feedbacks as are present at any specified time t must perforce respond to the entire reference signal then prevalent.

Linear algebra (Eq. 21) confirms the results in Eqs. (19, 20) and thus demonstrates Eq. (1):

$$\begin{aligned} & E_t \coloneqq R_t + b_t \coloneqq R_t + f_t E_t \\ \Rightarrow \quad & R_t = E_t - b_t = E_t - f_t E_t = E_t(1 - f_t) \\ \Rightarrow \quad & E_t = R_t / (1 - f_t) = R_t A_t \\ \Rightarrow \quad & A_t = E_t / R_t = (1 - f_t)^{-1}. \end{aligned} \tag{21}$$

Since the signal transits the feedback loop infinitely, A_t is the sum of a convergent infinite geometric series whose common ratio at time t is the feedback fraction f_t. Eq. (22) gives the partial sum $(E_t)_n$ of the first n terms, for constant R_t. Then Eq. (23) is the product of Eq. (22) and f_t.

$$(E_t)_n = {f_t}^0 R_t + {f_t}^1 R_t + {f_t}^2 R_t + \cdots + {f_t}^{n-1} R_t. \tag{22}$$

$$f_t (E_t)_n = f_t R_t + {f_t}^2 R_t + {f_t}^3 R_t + \cdots + {f_t}^n R_t. \tag{23}$$

Since all but R_t in Eq. (22) and ${f_t}^n R_t$ in Eq. (23) cancel, Eq. (24) = Eq. (22) − Eq.(23):

$$\begin{array}{rcl} (E_t)_n & = & R_t + \quad f_t R_t + {f_t}^2 R_t + {f_t}^3 R_t + \cdots + {f_t}^{n-1} R_t \\ -\,[f_t (E_t)_n & = & \qquad\quad f_t R_t + {f_t}^2 R_t + {f_t}^3 R_t + \cdots + {f_t}^{n-1} R_t \quad + {f_t}^n R_t] \\ \hline = \ (1 - f_t)(E_t)_n & = & R_t - {f_t}^n R_t \ = \ R_t (1 - {f_t}^n). \end{array} \tag{24}$$

The ratio of Eq. (24) and $(1-f_t)$ is $(E_t)_n$ (Eq. 25), whereupon, under the convergence criterion $|\,f_t\,| < 1$, Eq. (26) follows.

$$(E_t)_n = R_t(1-{f_t}^n)/(1-f_t). \tag{25}$$

$$n \to \infty \;\; \Rightarrow \;\; (1-{f_t}^n) \to 1 \;\; \Rightarrow \;\; R_t(1-{f_t}^n) \to R_t. \tag{26}$$

Accordingly, for $|\,f_t\,| < 1$, in Eq. (27) E_t is the product of R_t and the convergent infinite geometric series $\{{f_t}^0 + {f_t}^1 \ldots + {f_t}^\infty\}$, summing to $1/(1-f_t)$, again demonstrating Eq. (1).

$$E_t \;=\; R_t \frac{1}{(1-f_t)} \;=\; R_t + b_t \;=\; R_t + f_t E_t \;=\; R_t \sum_{i=1}^{\infty} f_t^i \;=\; R_t A_t \tag{27}$$

As is evident from Fig. 2, a feedback response occurs whenever a reference signal is present, and in response to that signal. With no reference signal, no feedback response arises. Accordingly, regardless of the shape of $E(R)$, $R_t = 0 \Rightarrow E_t = 0$, whereupon the point $(0, 0)$ always lies on the curve of $E(R)$.

4. The chief sensitivity-relevant feedbacks and the nonlinearities therein

The principal sensitivity-relevant temperature feedbacks (Table 1) were no less applicable in the absence of the NCGHGs than they are today. There are two important consequences. First, at any given time the great majority of the feedback response is attributable to emission temperature, which, in 1850, represented some 92% of the equilibrium temperature then prevalent. Secondly, no large change in the feedback fraction f_t is to be expected over time. Charney sensitivity consequential upon IPCC's values for the sensitivity-relevant feedbacks in Table 1 is derived from their sum via the feedback fraction f_2. IPCC's midrange estimates imply 2.6 K Charney sensitivity, almost 1 K below the 3.35 K midrange estimate implicit in results from the CMIP5 ensemble (Andrews et al. 2012). As will be seen, in reality Charney sensitivity is well below even 2.6 K. As Table 1 shows, in IPCC's understanding the midrange estimates of all sensitivity-relevant feedbacks other than water vapor effectively self-cancel.

TABLE 1 Current feedbacks based on IPCC (2013, p. 818, table 9.5 and p. 128, Fig. 1.2)

Temperature feedback	Lower bound	**Mid-range**	Upper bound	Timescale
Water vapor feedback $(\lambda_1)_2$	+1.3 W m^{-2} K^{-1}	**+1.6 W m^{-2} K^{-1}**	+1.9 W m^{-2} K^{-1}	Hours
Lapse rate feedback $(\lambda_2)_2$	−1.0 W m^{-2} K^{-1}	**−0.6 W m^{-2} K^{-1}**	−0.2 W m^{-2} K^{-1}	Hours
Cloud feedback $(\lambda_3)_2$	−0.4 W m^{-2} K^{-1}	**+0.3 W m^{-2} K^{-1}**	+1.1 W m^{-2} K^{-1}	Days
Surface albedo feedback $(\lambda_4)_2$	+0.2 W m^{-2} K^{-1}	**+0.3 W m^{-2} K^{-1}**	+0.4 W m^{-2} K^{-1}	Years
IPCC feedback sum $\lambda_2 = \sum_{i=1}^{4}(\lambda_i)_2$	+0.1 W m^{-2} K^{-1}	**+1.6 W m^{-2} K^{-1}**	+3.2 W m^{-2} K^{-1}	Years
IPCC feedback fraction $f_2 = \lambda_2 P_2$	[+0.0]	**+0.5**	[+1.0]	
IPCC system-gain factor $a_2 = 1/(1-f_2)$	[1.0]	**2.0**	[Undefined]	
IPCC implicit Charney sensi. $E_3 = \Delta R_2 a_2$	[1.0]	**2.6**	[∞]	

Some individual feedbacks $(\lambda_i)_2$ and hence λ_2, f_2, a_2, vary nonlinearly with temperature. To address the question whether significant nonlinearities in feedback response may arise within a policy-relevant timeframe, each feedback in Table 1 is now considered, together with the Planck feedback.

Water vapor and lapse-rate feedbacks: Among the sensitivity-relevant feedbacks, column water vapor feedback predominates. In accordance with the Clausius-Clapeyron equation, specific humidity is expected to grow with warming by an observed 7% K^{-1} (Wentz et al. 2007). However, there are strong reasons to expect a modest feedback response to changes in specific humidity. As with CO_2, so with water vapor, the forcing (here arising from feedback) is an approximately logarithmic function of the concentration. Furthermore, since ocean heat capacity is vast, negative feedbacks, such as the lapse-rate feedback and the earlier onset of tropical afternoon convection and cloud formation with warming, countervail to some degree against positive water-vapor feedback. In the lower troposphere, the only altitude at which specific humidity is rising as predicted (Kalnay et al. 1996, updated: Fig. 3), water vapor's spectral lines are near-saturated. Then, as specific humidity increases, only the far wings contribute to increased infrared absorption (Harde 2017). Absorption varies logarithmically with specific humidity: feedback response varies linearly with temperature.

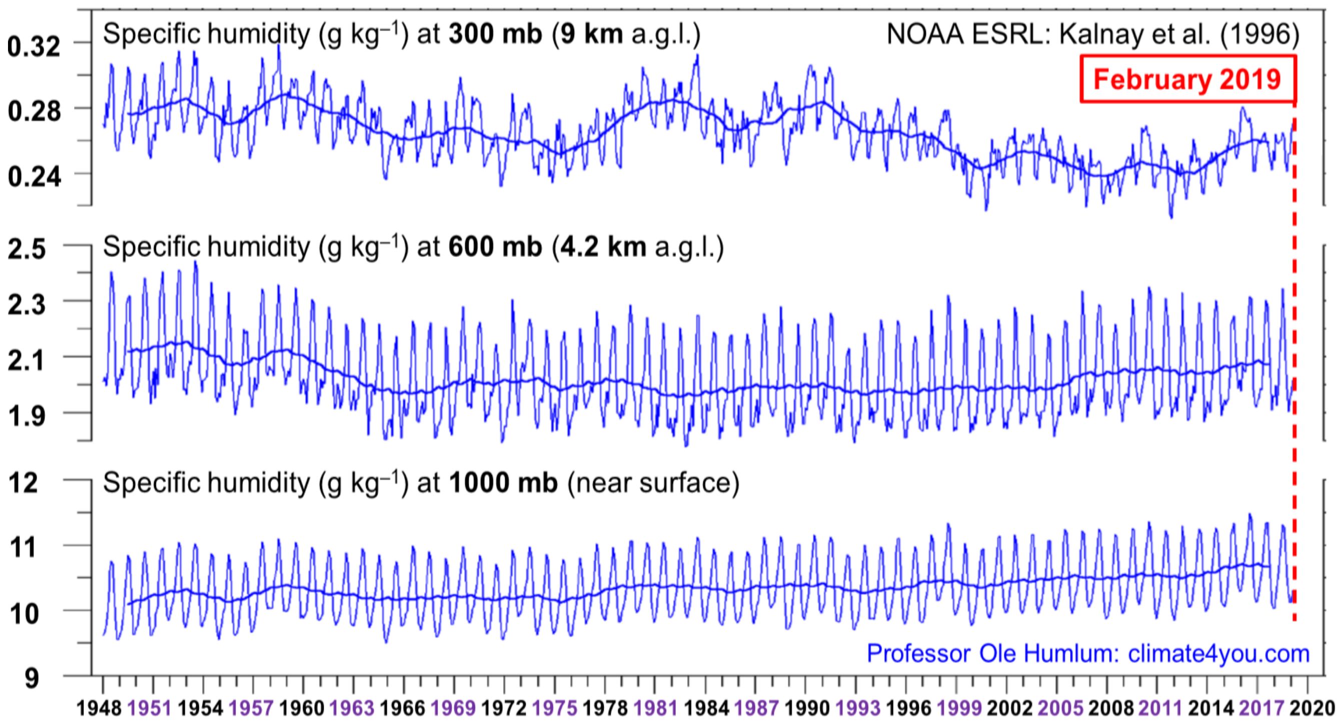

FIG. 3 Specific humidity (g kg^{-1}) at 300, 600 and 1000 mb

In GCMs, some 90% of the water vapor feedback is projected to arise in the tropical mid-troposphere, where, however, specific humidity has been declining for several decades (Fig. 3). Perhaps for this reason, most datasets (e.g. those underlying Figs. 3, 4b) do not show the "hot spot" in the tropical mid-troposphere (Fig. 4a), where the GCMs (Fig. 4a) predict that warming will be twice the warming at the tropical surface. Douglass et al. (2008) were among the first to draw attention to this discrepancy, which Paltridge et al. (2009) attributed to subsidence drying of the upper and mid-troposphere. Without that differential rate of warming, the water vapor feedback cannot, as is currently thought, double the reference sensitivity. Overstated water vapor feedback is, therefore, likely to be the chief physical reason for the currently-overstated system-gain factor implicit in the GCMs.

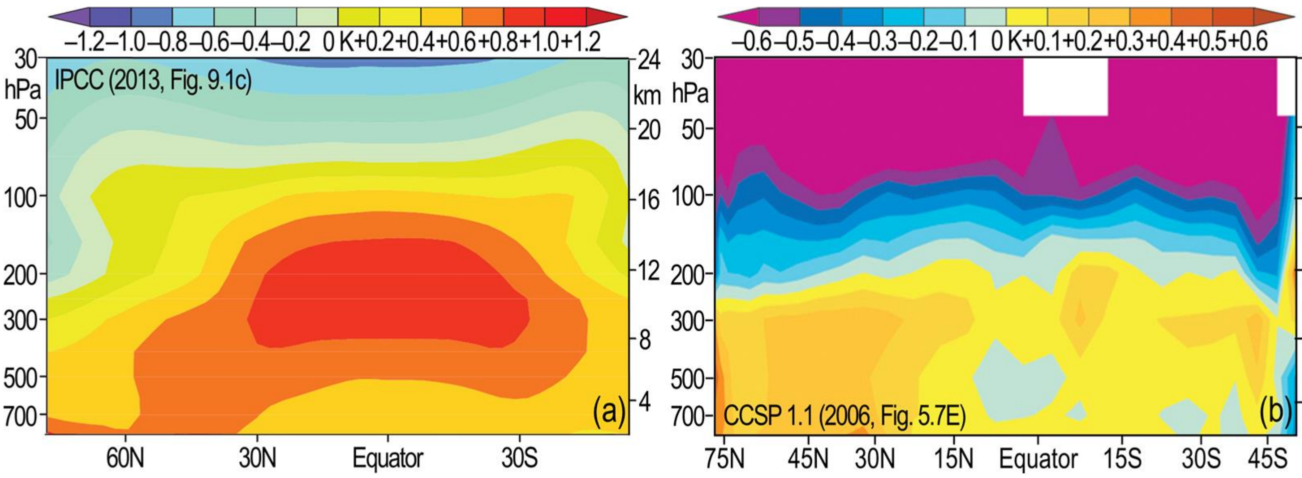

FIG. 4 GCMs' projected "hot spot"[28] (a) is absent in observational data[11] (b). Temperature anomalies (in Kelvin) are color-coded.

Though Santer et al. (2008) attempted to reconcile the models' projections of the mid-troposphere temperature profile with observations from 1979-1999, McKitrick et al. (2010) updated the datasets to 2009 and found that model-projected temperature trends in the lower as well as mid-troposphere exceeded observation twofold to fourfold, reporting that the differences were statistically significant at the 99% confidence interval. Christy (2010) noted that models projected that in the tropics the mid-troposphere would warm 1.4 times faster than the surface, while observations showed the surface warming 1.25 times faster than the mid-troposphere. While Thorne et al. (2011), in a meta-analysis, found no compelling evidence of disagreement between models and observations, Fu et al. (2011) found that the GCMs had overestimated the increase in static stability between the tropical mid- and upper troposphere. Though Sherwood & Nishant (2015) reported "robust tropospheric warming" in the tropics at a rate somewhat greater than at the surface, neither the RSS nor UAH satellite dataset shows greater warming in the tropical mid-troposphere than at the tropical surface. The debate continues.

Cloud feedback is subject to substantial uncertainty, but the net effect of increased cloud cover on temperature is one of cooling, in that the global shortwave cloud-albedo feedback exceeds the feedback due to retention of longwave radiation by clouds (Ramanathan et al. 1989). Due to a reduction in cloud cover, mean solar radiation at the Earth's surface increased by $0.16\ \mathrm{W\ m^{-2}\ yr^{-1}}$ from 1983-2001 (Pinker et al. 2005), accounting on its own for most of the global warming over the period (Monckton of Brenchley 2011). From 2002 onward, the cloud cover reappeared, leading to a 15-year standstill in global temperature. Since it is unlikely that a major change in global cloud cover will result from increases in reference temperature, GCMs treat cloud feedback as small.

Surface albedo feedback responds chiefly to changes in northern-hemisphere snow cover, which, however, has remained broadly constant during the period of satellite observation. For this reason, the GCMs regard it as small. As for the cryosphere, since nearly all remaining ice is at very high latitudes where the solar altitude is low, the contribution to surface albedo feedback from ice-melt is today negligible, as the following analysis demonstrates.

Earth's surface area is 4π x $(6378.2 \text{ km})^2$, or 511 million km^2. Minimum Arctic sea ice area is 4 million km^2, or 0.8% of the Earth's surface. Ice albedo is 0.66 (Pierrehumbert 2011). Assuming ocean-water albedo 0.06 if all the Arctic ice were to melt for the late-summer quarter, global mean albedo, now 0.3, would become $0.3 - 0.008\ (0.66 - 0.06)$, or 0.295. However, high-Arctic insolation is only one-quarter as powerful as mean terrestrial insolation, requiring division by 4; summer ice loss endures for at most 3 months, or half of the Arctic daylight period, requiring division by 2; and the Arctic has 75% cloud cover, requiring a further division by 4. Thus, Eq. (28) gives the revised global mean present-day albedo α_2 assuming total Arctic ice-melt in the late-summer quarter, which proves to be vanishingly different from today's albedo.

$$\alpha_2 = 0.3 - 0.008\frac{0.66 - 0.06}{4 \text{ x } 2 \text{ x } 4} = 0.2999. \tag{28}$$

The difference ΔR_0 in current emission temperature (Eq. 29), and in surface temperature ΔT_0 from the near-linear lapse rate, for TSI $S_0 = 1363.5 \text{ W m}^{-2}$ and the Stefan-Boltzmann constant σ, is:

$$\Delta T_0 = \Delta R_0 = [S_0(1 - 0.3)/4\sigma]^{¼} - [S_0(1 - 0.2999)/4\sigma]^{¼} = 0.009 \text{ K}. \tag{29}$$

This first-order analysis indicates that, even if the entire Arctic icecap were to melt for three months every summer, very little change in surface albedo feedback would arise. Therefore, even if that feedback were nonlinear, it is and, in foreseeable modern conditions, will remain too small to be significant. This conclusion is consistent with the findings of two recent evaluations of snow-cover feedbacks in current climate models: Rosenblum & Eisenman (2017) and Connolly et al. (2019).

Planck feedback, not separately listed by IPCC and not shown in Table 1, is likewise *de minimis*. Assuming constant insolation at 1363.5 W m^{-2} and constant albedo at 0.3, giving mean emission-altitude flux density of 238.6W m^{-2}, and assuming equilibrium temperature $E_1 = 287.55$ K in 1850, the Planck parameter P_1 was $287.55/(4 \text{ x } 238.6) = 0.301$; for 2011, P_2 was $288.5/954.4 = 0.302$; at 2xCO_2 it would be $(288.5 + 3.35)/954.4 = 0.306$. The three ratios are near-identical. Here, too, the change over time is small.

Since significant nonlinearities in the response to any individual temperature feedback are not to be expected, the feedback regime is likely to be approximately linear and time-invariant. Nevertheless, various exponential-growth scenarios will now be studied and their plausibility assessed.

5. Evolution of the equilibrium-sensitivity function $E(R)$

At each successive time t at which radiative equilibrium prevails, there subsists a distinct absolute value of the system-gain factor A_t reflecting the entire feedback response b_t to reference sensitivity R_t and the consequent modification of equilibrium sensitivity E_t by the feedback processes then prevalent. Four illustrative equilibria in the evolution of climate will be studied:

$t = 0$ at emission temperature R_0, before accounting for any forcing or feedback;
$t = 1$ in 1850, the date at which IPCC currently takes the industrial era as commencing;
$t = 2$ in 2011, the date to which climate-relevant data were updated for IPCC (2013); and
$t = 3$ upon a radiative forcing equivalent to a CO_2 doubling compared with 2011.

Direct perturbations in the concentration of the NCGHGs (chiefly CO_2, CH_4, O_3 and N_2O) are treated as radiative forcings, while consequential forcings arising from perturbations in the burden of the condensing greenhouse gas water vapor are counted among the temperature feedbacks. Such NCGHG feedbacks, including the CO_2 feedback, are overlooked here. They are subject to very large uncertainty. IPCC (2013, p. 818, table 9.5) omits them from its list of sensitivity-relevant feedbacks (Table 1).

As a first step, values of R_t will be derived as the basis for deriving E_t under various evolutionary scenarios. Here and throughout, temperatures will be given to the nearest 0.05 K.

For $t = 0$, emission temperature R_0 is the starting-point. Given present-day total solar irradiance S_0 ($= 1363.50$ W m^{-2}), mean planetary albedo $\alpha = 0.31$ (Soden & Held 2006; *cf.* Loeb 2009) and the Stefan-Boltzmann constant $\sigma = 5.6704 \times 10^{-8}$ W m^{-2} K^{-4}, R_0 is given by Eq. (30).

$$R_0 = [S_0(1 - \alpha)/(4\sigma)]^{1/4} = 254.70 \text{ K} \tag{30}$$

With no greenhouse gases and before allowing for feedback, R_0 would prevail at the Earth's surface. With NCGHG-driven warming as well as feedback processes such as the water vapor feedback, the effective emission altitude has risen and is now in the mid-troposphere. Climatology does not currently make allowance for Hölder's inequalities in deriving R_0: that question will be addressed later.

For $t = 1$ in 1850, the point (R_1, E_1) is well constrained, providing a datum for all the scenarios that follow. Equilibrium temperature E_1 was 287.55 K that year. Implicit reference sensitivity ΔR_0 to the NCGHGs was $0.25(287.55 - 252 \text{ K}) = 8.90$ K (Lacis et al. 2010). Alternatively, the anthropogenic CO_2 forcing of 1.68 W m^{-2} from 1850-2011 represented 75% of the 2.49 W m^{-2} net period anthropogenic radiative forcing (the midrange 2.29 W m^{-2} in IPCC 2013, fig. SPM.5, with 0.2 W m^{-2} added to correct IPCC's overstatement of the negative aerosol forcing). Then, for a 30 W m^{-2} total CO_2 forcing to date (Schmidt et al. 2010), anthropogenic forcing ΔQ_0 was $(30 - 1.68)/0.75$, or 37.76 W m^{-2}. Reference sensitivity $\Delta R_0 = \Delta Q_0 P_1$ was thus $37.76 \times 0.31 = 11.7$ K. A fair midrange estimate of ΔR_0 is the mean of these two estimates: i.e., $(8.9 + 11.7)/2$, or 10.30 K. Then reference temperature R_1, the sum of emission temperature R_0 and reference sensitivity ΔR_0 to the pre-industrial NCGHGs, was 265 K in 1850. In the presence of the pre-industrial NCGHGs, equilibrium temperature E_1 ($= 287.55$ K) was the difference between today's global mean surface temperature T_S ($= 288.4$ K: Morice et al. 2012) and the 0.85 K least-squares linear-regression trend on the HadCRUT4 data from 1850-2018. The climate was approximately at equilibrium in 1850: there was to be no trend in global temperature for 80 years. Since uncertainties in R_1, E_1 are quite small, and since anthropogenic perturbation had had little effect by 1850, A_1 ($= E_1/R_1$) was equal to 1.085 in that year.

For $t = 2$ in 2011, net midrange anthropogenic radiative forcing ΔQ_1 is given as $2.29\ \mathrm{W\,m^{-2}}$ (IPCC 2013, fig. SPM.5); but many authors (e.g. Seifert et al. 2015; Stevens 2015; Fiedler et al. 2017; Sato et al. 2018) find the aerosol forcing less negative than IPCC (2013). Applying a $0.2\ \mathrm{W\,m^{-2}}$ adjustment (Armour 2017: for a discussion, see Lewis & Curry 2018), the midrange estimate of ΔQ_1 rises to $2.49\ \mathrm{W\,m^{-2}}$. Thus, ΔR_1, the product of ΔQ_1 and the Planck parameter $P_2 = 0.31\ \mathrm{K\,W^{-1}\,m^2}$, was 0.75 K in 2011, so that reference temperature R_2, the sum of R_1 and ΔR_1, was 265.75 K.

For $t = 3$ following a radiative forcing $\Delta Q_2 = 3.45\ \mathrm{W\,m^{-2}}$ equivalent to the forcing from doubled CO_2 (half of the mean of the 4xCO_2 forcings found in 15 CMIP5 models listed in Andrews 2012, table 1), $\Delta R_2 = P_2\,\Delta Q_2 = 1.05$ K. Then reference temperature R_3, the sum of R_2 and ΔR_2, would be 266.80 K. Since the mean midrange Charney sensitivity $(\Delta E_2)_M$ in the same models (*ibid.*) was 3.35 K, the models' midrange system-gain factor $(A_M)_3$ implicit in the CMIP5 outputs is $(\Delta E_M)_2/\Delta R_2$, or 3.2.

Table 2 summarizes the evolution of reference temperature R_t.

TABLE 2 Evolution of midrange reference temperature R_t (to the nearest 0.05 K).

Emission temperature	Pre-industrial	In 1850	1850-2011	In 2011	2011-2xCO_2	At 2xCO_2
R_0	ΔR_0	R_1	ΔR_1	R_2	ΔR_2	R_3
254.70 K	10.30 K	265.00 K	0.75 K	265.75 K	1.05 K	266.80 K

6. Five models of the equilibrium-sensitivity response function $E(R)$

Since the shape of $E(R)$ is not known, values of $R, \Delta R$ in Table 2 will be deployed in five illustrative evolutions consistent with empirical data; four exponential-growth models and a linear-growth model. It will become apparent that current climate-sensitivity estimates are excessive, since they imply a disproportionately large feedback fraction in response to both natural and anthropogenic greenhouse warming compared with the feedback fraction in response to emission temperature. Model 1 assumes that current midrange Charney-sensitivity estimates are correct. Model 2 assumes that the feedback response grows at 7% per Kelvin of reference temperature. Model 3 assumes that the current observationally-based implicit midrange estimate of the anthropogenically-forced warming from 1850-2011 is correct. Model 4 assumes that a zero temperature implies a zero feedback response. Model 5 is a linear-growth model.

Model 1 is derived from current midrange Charney-sensitivity estimates. $E(R)$ is taken as an exponential-growth function derived from points (R_1, E_1), (R_3, E_3), where R_1, R_3 are as in Table 2, $E_1 = 287.55$ K and E_3 is derived from the CMIP5 system-gain factor $(A_M)_3$ via Eq. (31).

$$E_3 = E_1 + (A_M)_3(\Delta R_1 + \Delta R_2) = 293.30\ \mathrm{K}. \quad (31)$$

The shape of a unique exponential-growth function is derivable from any two specified points on the curve of the function. Here, on an exponential curve $E_t = k_1 \exp(k_2 R_t)$, the constants k_1, k_2 of exponentiality are derived from points (R_1, E_1), (R_3, E_3). Solving the simultaneous equations (32, 33) by way of Eqs. (34, 35) yields the constants k_1, k_2 (Eqs. 36, 37). Then the point-slope s_t at any point (R_t, E_t) is the first derivative (Eq. 38) of the function at that point, while Eq. (39) gives the slope a_t of the secant between points $(R_{t-1}, E_{t-1}), (R_t, E_t)$.

$$E_1 = k_1 \exp(k_2\ R_1); \qquad (32)$$

$$E_3 = k_1 \exp(k_2\ R_3). \qquad (33)$$

$$E_1/E_3 = \exp(k_2 R_1 - k_2 R_3) = \exp[k_2(R_1 - R_3)] \qquad (34)$$

$$\ln(E_1/E_3) = k_2(R_1 - R_3) \qquad (35)$$

$$k_2 = \ln(E_1/E_3)\,/(R_1 - R_3) = 0.0110; \qquad (36)$$

$$k_1 = E_1 \exp(-k_2 R_1) = 15.4883. \qquad (37)$$

$$s_t = k_1 k_2 \exp(k_2 R_t); \qquad (38)$$

$$a_t = (E_t - E_{t-1})/(R_t - R_{t-1}) \qquad (39)$$

Table 3 gives reference temperatures R_t, equilibrium temperatures E_t, feedback responses b_t, feedback fractions f_t, system-gain factors A_t, point-slopes s_t and secant-slopes a_t for model 1.

TABLE 3 Results from model 1

Emiss. temp.		Pre-industrial		In 1850		1850-2011		In 2011		2011 to $2xCO_2$		At $2xCO_2$	
R_0	254.70 K	ΔR_0	10.30 K	R_1	265.00 K	ΔR_1	0.75 K	R_2	265.75 K	ΔR_2	1.05 K	R_3	266.80 K
E_0	256.70 K	ΔE_0	30.85 K	E_1	287.55 K	ΔE_1	2.40 K	E_2	289.95 K	ΔE_2	3.40 K	E_3	293.30 K
b_0	2.00 K	Δb_0	20.55 K	b_1	22.55 K	Δb_1	1.65 K	b_2	24.20 K	Δb_2	2.35 K	b_3	26.50 K
$\boldsymbol{f_0}$	**0.0078**	$\boldsymbol{\Delta f_0}$	**0.6663**	f_1	0.0784	Δf_1	0.6858	f_2	0.0834	Δf_2	0.6889	f_3	0.0904
A_0	1.0078	ΔA_0	2.9966	A_1	1.0851	ΔA_1	3.1831	A_2	1.0910	ΔA_2	3.2149	A_3	1.0994
s_0	2.8297	(a_0)	(2.9966)	s_1	3.1700	(a_1)	(3.1831)	s_2	3.1963	(a_2)	(3.2149)	s_3	3.2335

At $t = 0$, the feedback response b_0 to the 254.70 K emission temperature R_0 is only 2.00 K, so that equilibrium temperature E_0 before accounting for any NCGHGs is 256.70 K and the feedback fraction f_0 $(= b_0/E_0)$ is just 0.0078. However, the feedback response Δb_0 to the 10.30 K reference sensitivity ΔR_0 driven by warming from the pre-industrial NCGHGs is 20.55 K. The feedback fraction Δf_0 $(= \Delta b_0/\Delta R_0)$ is 0.6663, exceeding f_0 by two orders of magnitude. Any such outcome is in practice impossible: there is no legitimate theoretical or empirical reason to suppose that the presence of the NCGHGs altered the pre-existing feedback regime so drastically as to increase the feedback fraction 85-fold. Therefore, an exponential-growth model dependent upon the assumption that anything like the current projections of Charney sensitivity are correct is untenable.

Another significant conclusion from this experiment is that the feedback fraction cannot approach unity, as is currently thought (see e.g. Schlesinger 1985; Roe, 2009). Once it is appreciated that feedback responds not only to perturbations but also to emission temperature, the notion of a “tipping-point” beyond which runaway feedbacks may rapidly and uncontrollably drive up global temperature becomes insupportable.

Model 2 assumes that the feedback response b_t, dependent chiefly upon the water-vapor feedback, will grow exponentially at 7% per Kelvin of reference temperature R_t in line with the currently-projected 7% K^{-1} Clausius-Clapeyron growth in specific humidity with temperature (Wentz et al. 2007). In reality, there is an approximately logarithmic relation between change in specific humidity and change in water-vapor feedback forcing, implying an approximately linear temperature response to the water-vapor feedback. Nevertheless, model 2 takes $E(R)$ as an exponential-growth function derived from points (R_1, E_1), (R_3, E_3), where R_1, R_3 are as in Table 2; E_1 is the observed global mean surface temperature in 1850; and E_3 is the sum of R_3 and b_3 as derived in Eq. (40).

$$E_3 = R_3 + b_3 = R_3 + b_1(1.07^{\Delta R_1 + \Delta R_2}) = 266.80 + 25.50 = 292.25 \text{ K}. \tag{40}$$

Calculation proceeds as in model 1. Eqs. (41, 42) give the constants k_1, k_2 of exponentiality.

$$k_2 = \ln(E_1/E_3)/(R_1 - R_3) = 0.0090; \tag{41}$$

$$k_1 = E_1 \exp(-k_2 R_1) = 26.1495. \tag{42}$$

Table 4 gives reference temperatures R_t, equilibrium temperatures E_t, feedback responses b_t, feedback fractions f_t, system-gain factors A_t, point-slopes s_t and secant-slopes a_t for model 2.

TABLE 4 Results from model 2

Emiss. temp.		Pre-industrial		In 1850		1850-2011		In 2011		2011 to $2xCO_2$		At $2xCO_2$	
R_0	254.70 K	ΔR_0	10.30 K	R_1	265.00 K	ΔR_1	0.75 K	R_2	265.75 K	ΔR_2	1.05 K	R_3	266.80 K
E_0	261.95 K	ΔE_0	25.60 K	E_1	287.55 K	ΔE_1	1.95 K	E_2	289.50 K	ΔE_2	2.75 K	E_3	292.25 K
b_0	7.25 K	Δb_0	15.30 K	b_1	22.55 K	Δb_1	1.20 K	b_2	23.75 K	Δb_2	1.70 K	b_3	25.50 K
$\mathbf{f_0}$	**0.0277**	$\mathbf{\Delta f_0}$	**0.5974**	f_1	0.0784	Δf_1	0.6169	f_2	0.0821	Δf_2	0.6200	f_3	0.0872
A_0	1.0285	ΔA_0	2.4841	A_1	1.0851	ΔA_1	2.6105	A_2	1.0894	ΔA_2	2.6318	A_3	1.0955
s_0	2.3701	(a_0)	(2.4841)	s_1	2.6016	(a_1)	(2.6105)	s_2	2.6194	(a_2)	(2.6318)	s_3	2.6444

As with model 1, model 2 is in practice impossible. The feedback response to the 254.7 K emission temperature is only 7.25 K; yet the feedback response to the 10.3 K sensitivity to the pre-industrial NCGHGs is 15.3 K. The feedback fraction Δf_0 in response to the pre-industrial NCGHGs exceeds the feedback fraction f_0 in response to emission temperature 22-fold. Once again, there is no plausible physical explanation for any such sudden increase in the feedback fraction.

Model 3 represents an exponential growth function $E(R)$ derived both from the well-constrained reference and equilibrium temperatures at point (R_1, E_1) in 1850 and from the current midrange projected values of those temperatures at point (R_2, E_2) in 2011, derived from the projected net anthropogenic forcing from 1850-2011 in IPCC (2013, fig. SPM.5). Since the climate was not in radiative equilibrium in 2011, the 0.75 K observed warming from 1850-2011 was not an equilibrium sensitivity; and the anthropogenic fraction of observed warming is unknown (Legates et al., 2015). To overcome these difficulties, an energy-balance model (Gregory 2004, Lewis & Curry 2018) was used.

To derive the temperature E_2 that would have prevailed if the climate had been in radiative equilibrium in 2011, one must allow for the change ΔN_{t-1} in the estimated top-of-atmosphere net radiative imbalance N_t at time $t = 2$ in 2011, assuming that the change Δq_{t-1} in net outgoing radiation consequent upon the net anthropogenic radiative forcing ΔQ_{t-1} is linearly proportional to reference sensitivity ΔR_{t-1}. In Eq. (43), the temperature-feedback parameter $(\lambda_F)_t$ is the growth in the net outgoing radiative flux Δq_{t-1} per Kelvin of surface warming ΔR_{t-1}. Internal variability is ignored. By conservation of energy, Eq. (44) gives the radiative imbalance ΔN_{t-1}. Then the feedback parameter $(\lambda_F)_t$, derived from Eqs. (43, 44), is given by Eq. (45).

$$(\lambda_F)_t = \Delta q_{t-1}/\Delta R_{t-1}. \tag{43}$$

$$\Delta N_{t-1} = \Delta Q_{t-1} - \Delta q_{t-1}. \tag{44}$$

$$(\lambda_F)_t = (\Delta Q_{t-1} - \Delta N_{t-1})/\Delta R_{t-1}. \tag{45}$$

Where ΔQ_{t-1} is the radiative forcing and ΔE_{t-1} is equilibrium sensitivity, once the climate system has settled to equilibrium (i.e., where $\Delta N_{t-1} = 0$), Eq. (46) yields the temperature feedback parameter $(\lambda_F)_t$, whereupon, by substitution in Eq. (45), Eq. (47) yields equilibrium sensitivity ΔE_{t-1}. Since reference sensitivity ΔR_{t-1} is the product of the Planck sensitivity parameter P_t and the forcing ΔQ_{t-1}, Eq. (47) is recast as Eq. (48) to give the slope a_1 of the secant from 1850-2011. It is this slope that climatology takes to be its system-gain factor.

$$(\lambda_F)_t = \Delta Q_{t-1}/\Delta E_{t-1}, \tag{46}$$

$$\Delta E_{t-1} = \Delta Q_{t-1} \frac{\Delta R_{t-1}}{\Delta Q_{t-1} - \Delta N_{t-1}}. \tag{47}$$

$$a_1 = \frac{E_2 - E_1}{R_2 - R_1} = \frac{\Delta E_1}{\Delta R_1} = \frac{\Delta E_1}{\Delta Q_1 P_2} = \frac{\Delta Q_1}{\Delta Q_1 - \Delta N_1}. \tag{48}$$

Anthropogenic forcing ΔQ_1 from 1850-2011 and radiative imbalance ΔN_1 to 2010 are subject to large uncertainties. Midrange estimates are $\Delta Q_1 = 2.49\ \mathrm{W\ m^{-2}}$ ($2.29\ \mathrm{W\ m^{-2}}$ IPCC 2013, fig. SPM.5, adjusted for a $0.2\ \mathrm{W\ m^{-2}}$ overestimate of negative aerosol forcing, based on Armour 2017), and $\Delta N_1 = 0.6\ \mathrm{W\ m^{-2}}$ (Smith et al. 2015). Then the midrange estimate of the slope a_1 of the industrial-era secant from 1850-2011 (Eq. 48) is 1.3175, implying period equilibrium sensitivity $\Delta E_1 = 1.0$ K, so that equilibrium temperature E_2 ($= E_1 + \Delta E_1$) was 288.55 K. However, the true system-gain factor A_2 ($= E_2/R_2 = 288.55/265.75$) was 1.086, scarcely above $A_1 = 1.085$. Here, $k_1 = 0.0045$ and $k_2 = 85.5720$. Table 5 gives results for model 3.

TABLE 5 Results from model 3

Emiss. temp.		Pre-industrial		In 1850		1850-2011		In 2011		2011 to 2xCO_2		At 2xCO_2	
R_0	254.70 K	ΔR_0	10.30 K	R_1	265.00 K	ΔR_1	0.75 K	R_2	265.75 K	ΔR_2	1.05 K	R_3	266.80 K
E_0	274.30 K	ΔE_0	13.25 K	E_1	287.55 K	ΔE_1	1.00 K	E_2	288.55 K	ΔE_2	1.40 K	E_3	289.95 K
b_0	19.60 K	Δb_0	2.95 K	b_1	22.55 K	Δb_1	0.25 K	b_2	22.80 K	Δb_2	0.35 K	b_3	23.15 K
$\boldsymbol{f_0}$	**0.0715**	$\boldsymbol{\Delta f_0}$	**0.2216**	f_1	0.0784	Δf_1	0.2410	f_2	0.0790	Δf_2	0.2441	f_3	0.0798
A_0	1.0770	ΔA_0	1.2847	A_1	1.0851	ΔA_1	1.3175	A_2	1.0858	ΔA_2	1.3229	A_3	1.0867
s_0	1.2547	(a_0)	(1.2847)	s_1	1.3152	(a_1)	(1.3175)	s_2	1.3197	(a_2)	(1.3229)	s_3	1.3261

Model 3 is less implausible than models 1-2, showing feedback responses of 19.6 K to the 254.70 K emission temperature, and of 2.95 K to the 10.3 K reference sensitivity to the pre-industrial NCGHGs. However, the feedback fractions are 0.0715 and 0.2216 respectively: even here, implausibly, the feedback fraction in response to the NCGHGs is thrice that in response to emission temperature. Model 3 also suggests that, on the basis of the 1.05 K industrial-era equilibrium sensitivity, Charney sensitivity would be only 1.4 K, not the CMIP5 models' currently-projected 3.35 K.

Model 4 correctly takes a zero temperature as entailing no feedback response; the y-intersect is 0. For an exponential-growth curve $E = R^x$ through points $(0,0)$, $(R_1, E_1) = (265.00, 287.55)$, Eq. (49) gives the exponent x, whereupon Eq. (50) gives E_t for any value of R_t.

$$x = \ln(E_1/R_1) = 1.0146. \tag{49}$$

$$E_t = R_t^x. \tag{50}$$

Table 6 gives results for model 4.

TABLE 6 Results from model 4

Emiss. temp.		Pre-industrial		In 1850		1850-2011		In 2011		2011 to $2xCO_2$		At $2xCO_2$	
R_0	254.70 K	ΔR_0	10.30 K	R_1	265.00 K	ΔR_1	0.75 K	R_2	265.75 K	ΔR_2	1.05 K	R_3	266.80 K
E_0	276.20 K	ΔE_0	11.35 K	E_1	287.55 K	ΔE_1	0.85 K	E_2	288.40 K	ΔE_2	1.15 K	E_3	289.55 K
b_0	21.50 K	Δb_0	1.05 K	b_1	22.55 K	Δb_1	0.10 K	b_2	22.65 K	Δb_2	0.10 K	b_3	22.75 K
$\boldsymbol{f_0}$	**0.0779**	$\boldsymbol{\Delta f_0}$	**0.0915**	f_1	0.0784	Δf_1	0.0918	f_2	0.0785	Δf_2	0.0918	f_3	0.0785
A_0	1.0845	ΔA_0	1.1007	A_1	1.0851	ΔA_1	1.1010	A_2	1.0852	ΔA_2	1.1011	A_3	1.0852
s_0	1.1004	(a_0)	(1.1007)	s_1	1.1010	(a_1)	(1.1010)	s_2	1.1010	(a_2)	(1.1011)	s_3	1.1011

Since the exponent x, calculated from the two reliable points $(0,0)$ and $(265.00, 287.55)$, is only 1.0146 (Eq. 49), model 4 is very close to linear. It is a plausible model, since the feedback fraction in response to the pre-industrial NCGHGs, at 0.0915, exceeds the feedback fraction 0.0779 in response to emission temperature by little more than one-sixth, which is not impossible.

Model 5 takes $E(R)$ as a linear function whose slope is equal to A_1 $(= E_1/R_1 = 1.085)$. Results for model 5 are almost identical to those for model 4, since the exponential-growth curve in model 4 is very close to linear. For this reason, it is legitimate to obtain approximate estimates of Charney sensitivity using an assumption of linearity in feedback response. Charney sensitivity in model 5, as in model 4, is 1.15 K. Table 7 gives results for model 5.

TABLE 7 Results from model 5

Emiss. temp.		Pre-industrial		In 1850		1850-2011		In 2011		2011 to $2xCO_2$		At $2xCO_2$	
R_0	254.70 K	ΔR_0	10.30 K	R_1	265.00 K	ΔR_1	0.75 K	R_2	265.75 K	ΔR_2	1.05 K	R_3	266.80 K
E_0	276.35 K	ΔE_0	11.20 K	E_1	287.55 K	ΔE_1	0.80 K	E_2	288.35 K	ΔE_2	1.15 K	E_3	289.50 K
b_0	21.70 K	Δb_0	0.90 K	b_1	22.55 K	Δb_1	0.05 K	b_2	22.60 K	Δb_2	0.10 K	b_3	22.70 K
$\boldsymbol{f_0}$	**0.0784**	$\boldsymbol{\Delta f_0}$	**0.0784**	f_1	0.0784	Δf_1	0.0784	f_2	0.0784	Δf_2	0.0784	f_3	0.0784
A_0	1.0851	ΔA_0	1.0851	A_1	1.0851	ΔA_1	1.0851	A_2	1.0851	ΔA_2	1.0851	A_3	1.0851
s_0	1.0851	(a_0)	(1.0851)	s_1	1.0851	(a_1)	(1.0851)	s_2	1.0851	(a_2)	(1.0851)	s_3	1.0851

Which model is preferable? Models 4, 5 have many advantages. Not the least of these is that, as expected, $R_t = 0 \Rightarrow E_t = 0$. All the other models imply, *per impossibile,* that a zero temperature will drive a positive feedback response. However, the chief advantage of models 4, 5 is that they take full account of the fact that feedback processes respond not only to anthropogenic perturbations in emission temperature but also to natural perturbations and also, most importantly, to emission temperature itself. The feedback response to emission temperature is, as it should be, larger than the feedback response to the pre-industrial NCGHG-driven warming, which is in turn larger than the feedback response to the smaller anthropogenic perturbation after 1850.

For comparison between the five models, Table 8 gives feedback responses $b_0, \Delta b_0$, feedback fractions $f_0, \Delta f_0$, feedback-fraction ratios $\Delta f_0/f_0$, system-gain factors A_3 and Charney sensitivities ΔE_2.

TABLE 8 Relationship between elevated ratios $\Delta f_0/f_0$ and elevated Charney sensitivities ΔE_2

	Model	b_0	Δb_0	f_0	Δf_0	$\Delta f_0/f_0$	A_3	ΔE_2
CMIP5 current ΔE_2:	1	2.00 K	20.55 K	**0.0078**	**0.6663**	**85**	1.0994	3.40 K
b_t + 1.07% K^{-1}:	2	7.25 K	15.30 K	**0.0277**	**0.5974**	**22**	1.0955	2.75 K
IPCC anth. forcing:	3	19.60 K	2.95 K	**0.0715**	**0.2216**	**3.1**	1.0867	1.40 K
$R = 0 \Rightarrow b = 0$:	4	21.50 K	1.05 K	**0.0779**	**0.0915**	**1.2**	1.0852	1.15 K
Linear:	5	21.70 K	0.90 K	**0.0784**	**0.0784**	**1.0**	1.0851	1.15 K

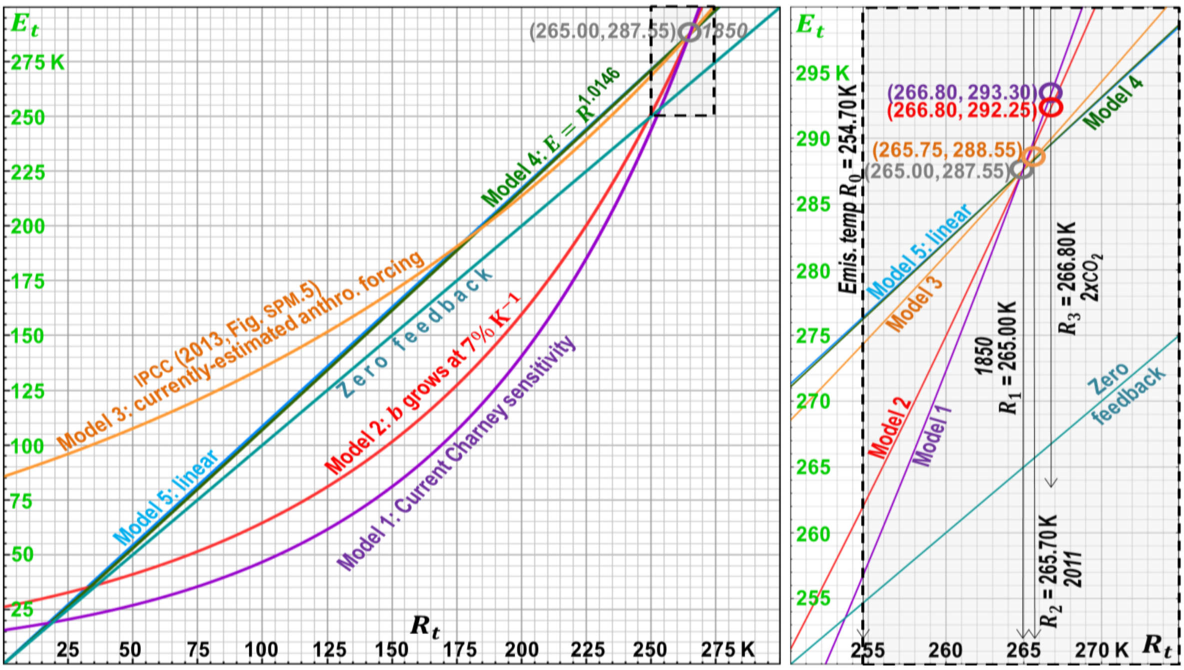

FIG. 5 Comparison of the five models of the evolution of $E(R)$ for R on $[250, 275]$ K. Models 1 (purple), 2 (red) and 3 (orange) are each generated from two points: the circled points in their colors and the common gray point $(265.00, 287.55)$ representing the position in 1850. In model 4 (green), the exponent $x = \ln(287.55)/\ln(265.70) = 1.0146$. Model 5 (pale blue) is the linear model. For comparison, the zero-feedback line $E = R$ is shown in turquoise. All five models appear linear across the interval R_t on $[250, 275]$ K (right panel). It is possible that the shape of the response function $E(R)$ is neither linear nor exponential. However, the fact that, owing to the dominance of emission temperature in the climate system, R_1 is more than 92% of E_1 strongly suggests that large departures from linearity in equilibrium-temperature response are not to be expected.

Even if Charney sensitivity were as little as 1.40 K in line with current midrange estimates of anthropogenic forcing and radiative imbalance assuming all industrial-era warming to be anthropogenic, as model 3 suggests, it is implausible that the feedback fraction in response to the pre-industrial NCGHGs would be as much as thrice that in response to emission temperature.

If Charney sensitivity is 1.15 K, as models 4, 5 suggest, the growth in the feedback response b in model 2 will be a plausible 0.5% per Kelvin of reference temperature, rather than the impossible 7% K^{-1} illustrated in model 2 (impossible because approximately logarithmic temperature response largely offsets exponential growth in specific humidity, giving a legitimate expectation of a near-linear response). The growth in the feedback fraction with temperature would likewise be plausible (Fig. 5).

The evolution of the feedback fraction f_t from emission temperature R_0 (= 254.70 K) to reference temperature R_1 (= 265.00 K) in 1850 (Fig. 6) reveals why it is that current projections of Charney sensitivity (model 1) are impossibly excessive. There is no physical basis for assuming that the ratio of the feedback fraction in response to the warming forced by the pre-industrial NCGHGs to the feedback fraction in response to emission temperature (the feedback-fraction ratio) will greatly exceed unity.

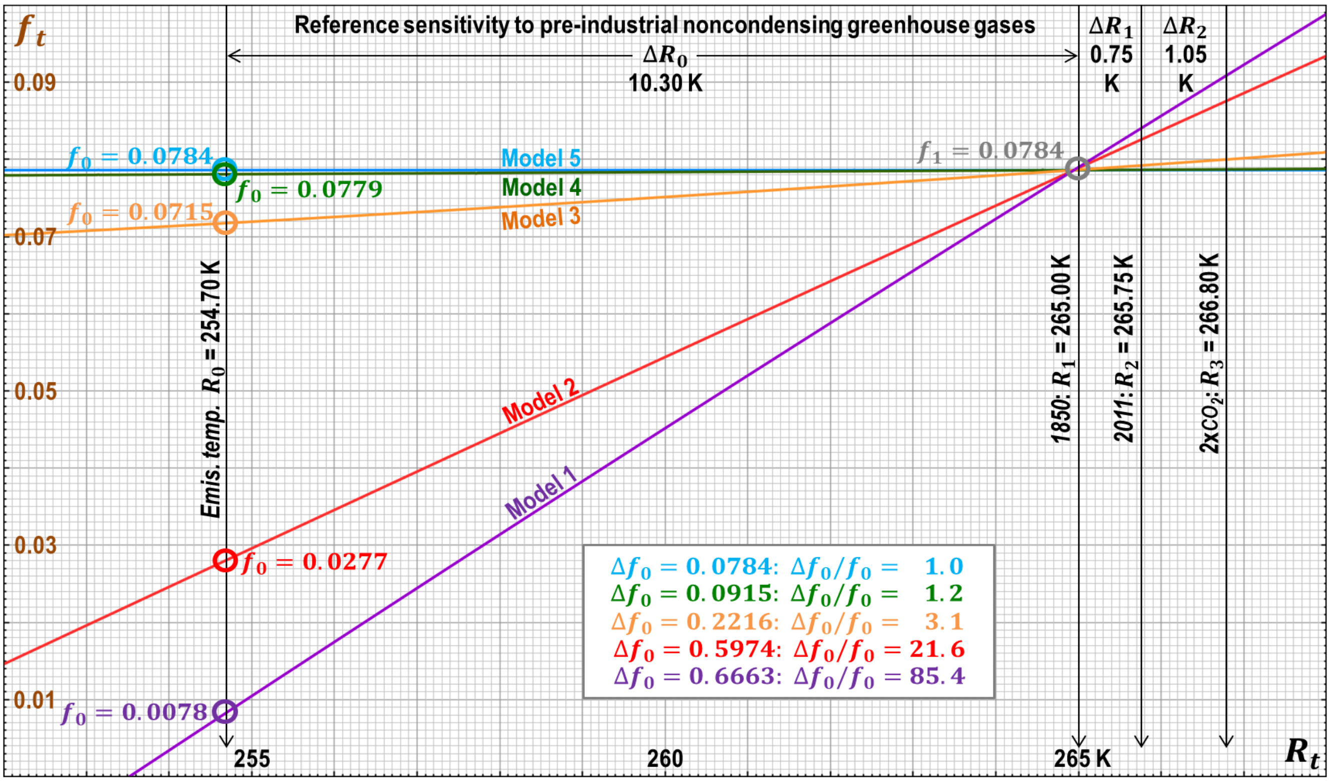

FIG. 6 Evolution of the feedback fraction f_t from emission temperature R_0 (= 254.7 K) to reference temperature R_1 (= 265.00 K) in 1850 and beyond.

Where $\Delta E_2 > 1.25$ K, the ratio of Δf_0 (= $\Delta b_0 / \Delta E_0$) to f_0 (= b_0 / E_0) becomes unrealistically large. Thus, models 1 and 2 are impossible and model 3 is implausible, while models 4-5 are realistic. As Fig. 7 shows, as the estimated Charney sensitivity rises above 1.4 K, the feedback-fraction ratio becomes so large as to imply a physically-unjustifiable growth in the feedback-fraction ratio. Since the sensitivity-altering temperature feedbacks in response to warming arising from increases in the atmospheric burden of the NCGHGs are precisely the same feedbacks that responded to the emission temperature consequent upon the fact that the Sun is shining, there is no reason to suppose that the feedback regime has changed or will change anything like as drastically as current equilibrium-sensitivity projections imply.

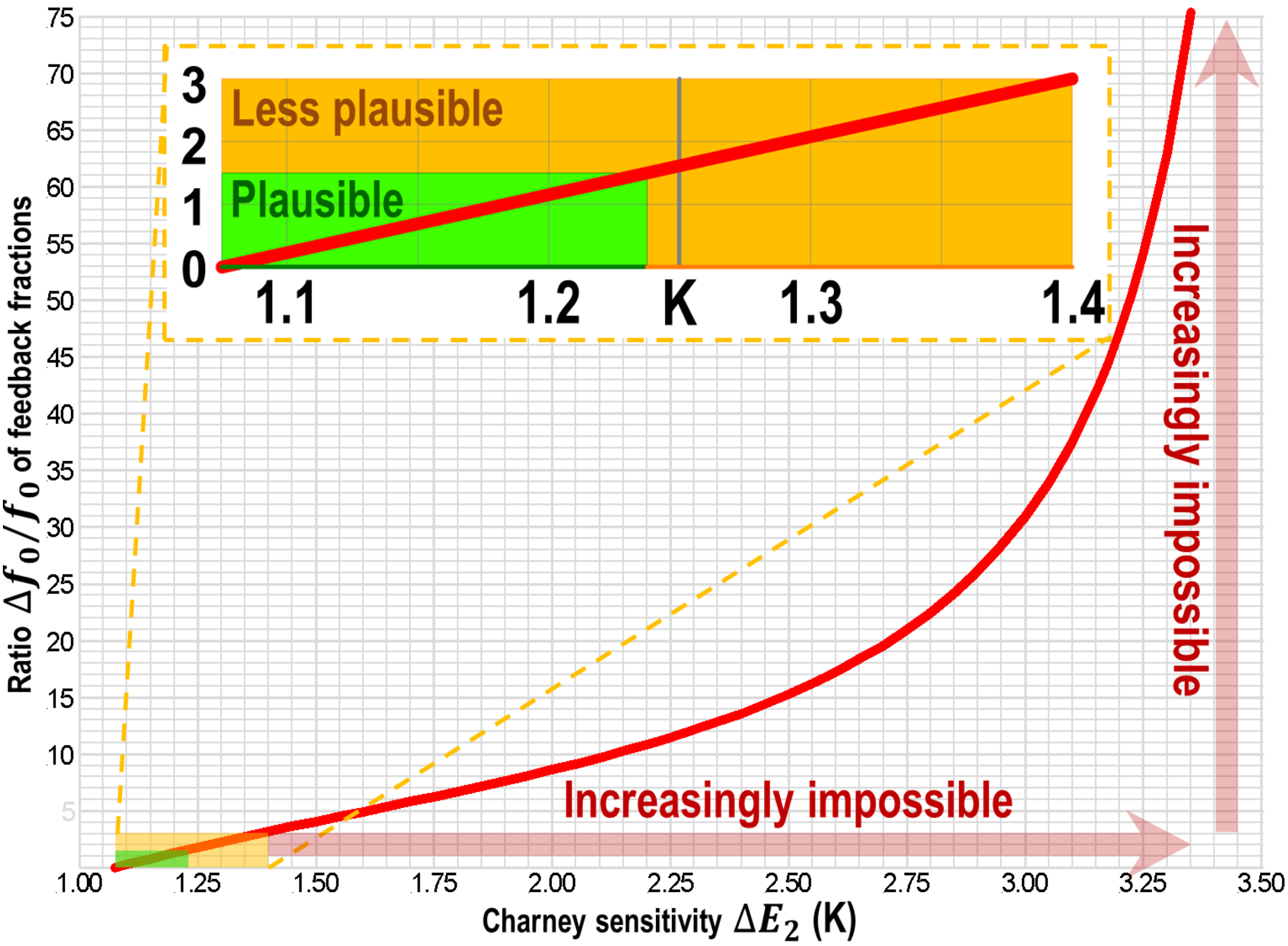

FIG. 7 The feedback-fraction ratio $\Delta f_0/f_0$, i.e., the ratio of the feedback fraction Δf_0 in response to reference sensitivity to the pre-industrial NCGHGs and the feedback fraction f_0 in response to emission temperature, for Charney sensitivity ΔE_2 on $[1.07, 3.35]$ K, for equilibrium temperature E_t an exponential-growth function $E(R)$ of reference temperature R_t. Green region: plausible sensitivities; orange region: implausible sensitivities. Beyond these, elevated feedback-fraction ratios and thus sensitivities are increasingly impossible.

Block diagrams such as Fig. 3 do not show the relative magnitudes of the contributions to reference and equilibrium temperatures. Fig. 8, based on model 4, is an attempt to remedy this defect. Scaled temperature responses to anthropogenic forcings and feedback responses thereto are visibly small compared with the temperature responses to natural forcings and minuscule against emission temperature and the feedback response thereto. It is for this reason that, particularly under modern conditions, large variations in the feedback regime in response to the small anthropogenic perturbation of global temperature are not to be expected. The high equilibrium sensitivities that are currently projected effectively misallocate the large feedback response to emission temperature, improperly attributing it to anthropogenic increases in the NCGHGs.

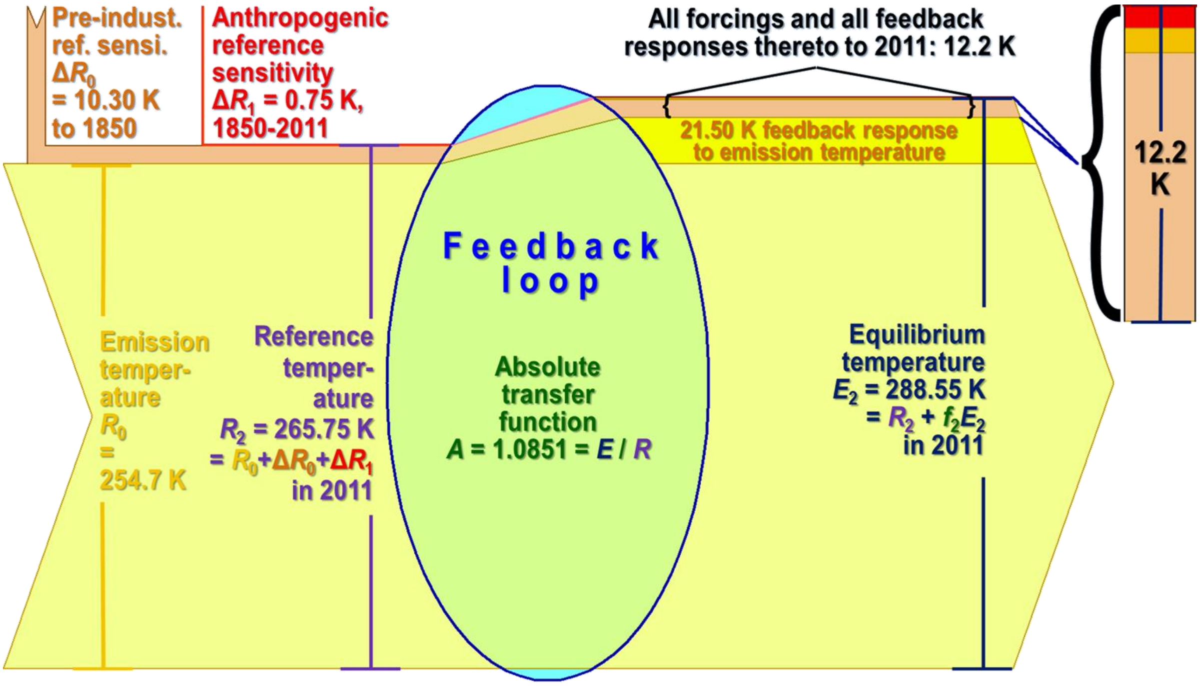

FIG. 8 Relative magnitudes of the contributions to reference temperature R_2 and to equilibrium temperature E_2 in 2011, assuming $E_t = g(R_t) = R_t^x$ (model 4). Dominance of emission temperature (pale yellow) and its feedback response (bright yellow) is visible.

7. Verification in the laboratory

In climate, individual temperature feedbacks cannot be measured. However, feedback theory (Bode, 1945: Fig. 9) applies no less to climate than to the electronic networks for which it was derived. Since states of a circuit can be directly measured more reliably than states of the climate, testing at two laboratories using circuits designed to represent features of the climate verified the theory outlined here.

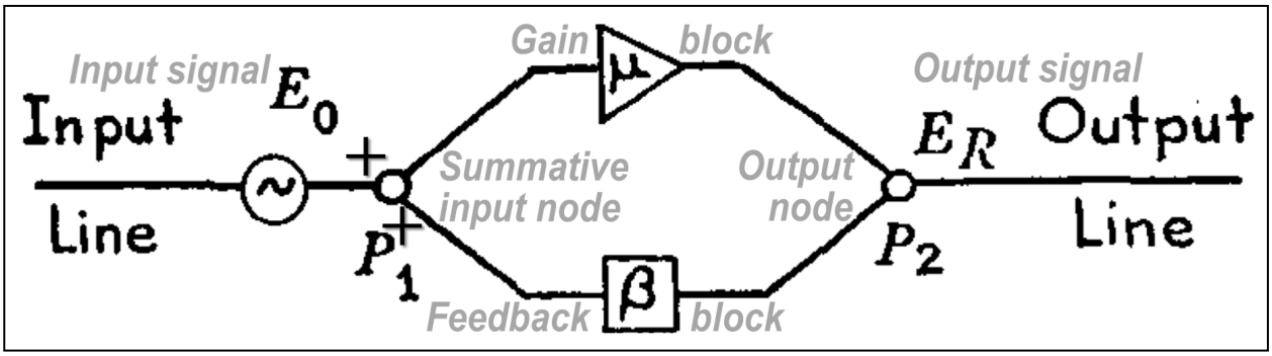

FIG. 9 A feedback amplifier with a μ gain block and a β feedback block (Bode 1945)

Based on a circuit built at the laboratory of an author (Whitfield) to simulate feedback loops electronically, a government laboratory built and ran a more sensitive rig. The input signal (E_0 in Bode 1945, p. *vii*), the open-loop gain factor μ and the feedback ratio β in Fig. 9 could be varied, whereupon the output signal (Bode's E_R) could be measured directly. The laboratory was given 23 sets of three values in four test groups and configured the circuit using each set, measuring the output signal in a temperature-controlled chamber.

After some months' delay owing to heat from the presence of the operator, which entailed revision of the inputs to yield the required precision without invalidating the tests, the laboratory reported. Results of all 23 tests, given in supplementary matter at S1 and reported by the laboratory at S2, agreed with the theory discussed here to a precision equivalent to 0.1 K. To overcome variances in the performance of individual components in the test circuit, each input value was measured to ensure that it was within tolerance. The result from test group 3 showed that, even without any amplification, i.e., where $\mu \coloneqq 1$ in Fig. 9, any output signal E drives a feedback response where feedbacks are present. Results for all test groups were as follows:

1. For f_3 ($= \mu\beta$) on $(f_{\text{mid}})_3 \pm 40\%$ (from Vial et al., 2013) and $\Delta R_2 = 1.16$ K (based on Myhre et al. 1998 and IPCC 2001), and even before correcting the error identified here, Charney sensitivity ΔE_2 falls on 2.3 [1.6, 3.6] K and not on the CMIP5 interval 3.35 [2.1, 4.7] K.
2. Where absolute input and output signals R_3, E_3 replace $\Delta R_2, \Delta E_2$ the interval of Charney sensitivities ΔE_2 narrows from 3 K to < 1 K, and the upper bound falls. From this experiment, current overstatements of the feedback factor f_t and the system-gain function A_t were first noticed.
3. Even where $\mu \coloneqq 1$ (i.e., the input signal is unamplified), the output signal exceeds it by the expected margin in the presence of positive feedback; and, where $\mu > 1$, the output signal does not much exceed the value for $\mu \coloneqq 1$. This experiment revealed the magnitude of the error of neglecting the feedback response to absolute temperatures.
4. After correction of climatology's error of definition, the magnitude and interval breadth of output responses to absolute system-gain functions A_t are small.

8. Uncertainties

The fact that feedback responds not only to anthropogenic reference sensitivity but also to reference sensitivity to the pre-industrial NCGHGs and also, most importantly, to emission temperature powerfully constrains the uncertainty as to the shape of the equilibrium-temperature response function $E(R)$. It has been demonstrated here that the steep exponential-growth scenarios implicit in current estimates of Charney sensitivity are impossible because they imply a feedback response to NCGHGs that exceeds by orders of magnitude the feedback response to emission temperature.

Thus, uncertainty in Charney sensitivity ΔE_2 is small, because even large uncertainties in absolute temperatures entail small uncertainties in their ratio, the absolute system-gain factor A. Furthermore, the Charney-sensitivity interval obtained via A (Eq. 1) falls on the near-linear region near the origin of the hyperbolic curve of Charney-sensitivity response to feedback fractions f_3 (Fig. 10).

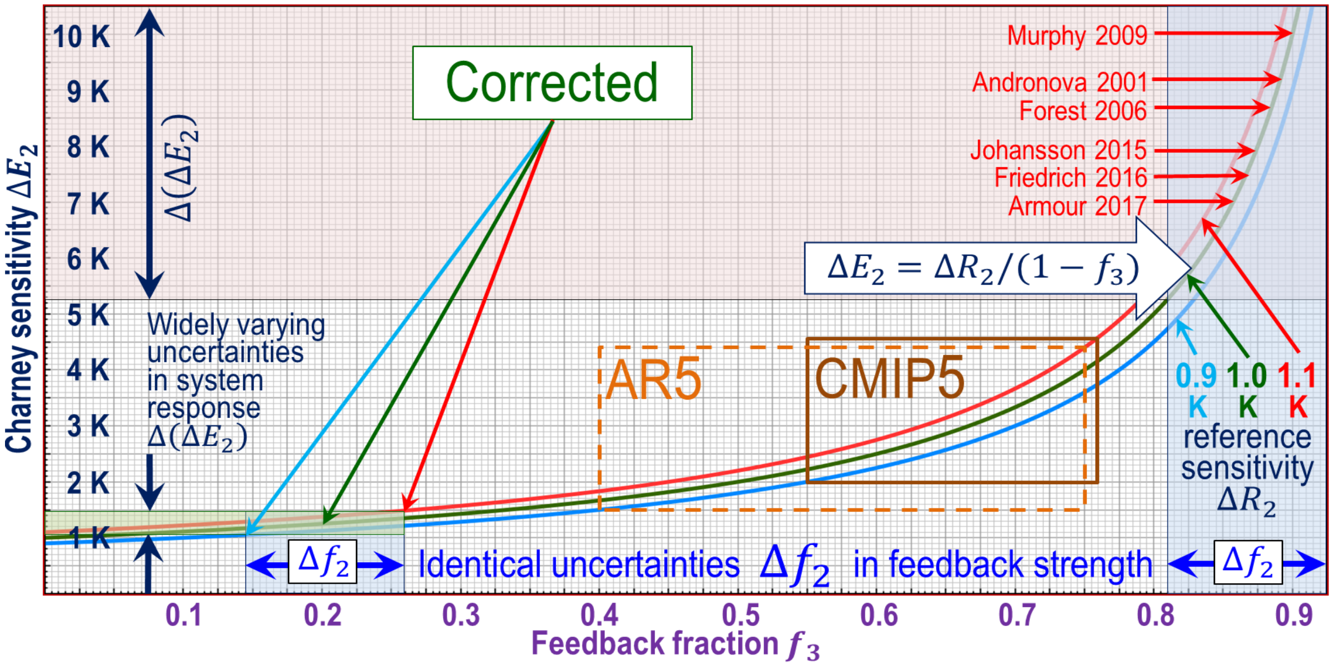

FIG. 10 The rectangular-hyperbolic response curves of Charney sensitivities ΔE_2 against feedback factors f_3 for reference sensitivity ΔR_2 on 1.0 K $\pm$ 10%. Identical uncertainties Δf_2 in f_3 generate broader uncertainty intervals $\Delta(\Delta E_2)$ in system response ΔE_2 as $f_3 \to 1$, since $\Delta(\Delta E_2)$ depends greatly on f_3 (Roe 2009, fig. 6). High-end predictions of ΔE_2 from six sources, the CMIP5 GCMs' interval [2.1, 4.7] K and the 2 σ interval 1.15 [1.10, 1.25] K from Eq. (1) are shown. Varying f_3 visibly makes very much more difference to ΔE_2 than varying ΔR_2, particularly where $f_3 \to 1$.

Since there is little uncertainty in deriving $A_3 = 1.085$, Charney sensitivity $\Delta E_2 = 1.15$ K. Though there are uncertainties in deriving $\Delta E_1, \Delta R_1$ for 2011, the values of both sensitivities are so small when set against E_1, R_1 (R_1 being > 350 times ΔR_1) that they barely perturb A.

Allowance should also be made for Hölder's inequalities between integrals in deriving emission temperature R_0. Integrating latitudinal temperatures on the dayside of an ice planet, the hemispheric mean temperature would be 240.6 K assuming today's insolation with uniform albedo 0.66. However, this value becomes 268.5 K assuming mean ocean surface albedo 0.06 in the ice-free tropics, since, to first-order approximation, at today's insolation one-third of the dayside surface area would be ice-free even before taking account of evapotranspiration and temperature feedbacks. Allowing for a nightside temperature of ~240 K (based on Merlis 2010), global mean emission temperature in the absence of greenhouse gases or of temperature feedbacks would be ~255 K, coincidentally near-identical with the value derived using a single global application of the Stefan-Boltzmann equation at today's albedo 0.3.

Industrial-era uncertainties, though greater than in 1850, barely affect $A_2 = 1.085$ in Eq. (1). An empirical campaign (Table 9), drawing upon ten authoritative sources for anthropogenic radiative forcing over various periods in the industrial era, established that in each of the ten cases A_2 fell on the interval [1.085, 1.088]. However, as cols. 7, 8 of Table 9 indicate, in the current method uncertainty is considerable: the upper bound of the partial system-gain factor a_2 is not 1.08 but 3.74. To the nearest 0.05 K, all ten data sources generate Charney sensitivity $\Delta E_2 = 1.15$ K (Col. 10).

TABLE 9 System-gain factors A_2 and Charney sensitivities ΔE_2 in the industrial era to 2011

		ΔQ_1	ΔN_1	ΔT_1	ΔR_1	ΔE_1	$\boldsymbol{\Delta E_1}$	a_2	$\boldsymbol{A_2}$	ΔE_2	$\boldsymbol{\Delta E_2}$
		− W m^{-2} −		K				Unitless		K	
Data source	Year	1	2	3	4	*5*	**6**	*7*	**8**	*9*	**10**
Miller 2014	2012	2.95	0.60	0.76	0.91	*2.88*	**0.95**	*1.04*	**1.085**	*1.11*	**1.15**
Myhre 2017	2016	3.10	0.60	0.84	0.96	*3.03*	**1.04**	*1.08*	**1.085**	*1.16*	**1.15**
IPCC AR5 2013	1980	1.25	0.40	0.38	0.39	*1.22*	**0.56**	*1.44*	**1.086**	*1.54*	**1.15**
Knutti 2002	2001	1.90	0.50	0.62	0.59	*1.86*	**0.84**	*1.43*	**1.086**	*1.53*	**1.15**
IPCC AR5 2013	1950	0.57	0.20	0.26	0.18	*0.56*	**0.40**	*2.27*	**1.086**	*2.42*	**1.15**
IPCC AR5 2013	**2011**	**2.29**	**0.60**	**0.76**	**0.71**	***2.24***	**1.03**	***1.45***	**1.086**	***1.55***	**1.15**
Haywood 2007	2006	1.93	0.50	0.68	0.60	*1.88*	**0.92**	*1.53*	**1.086**	*1.64*	**1.15**
IPCC AR4 2007	2005	1.60	0.50	0.67	0.50	*1.56*	**0.97**	*1.96*	**1.087**	*2.10*	**1.15**
Skeie 2011	2010	1.40	0.60	0.74	0.43	*1.37*	**1.30**	*2.98*	**1.088**	*3.19*	**1.15**
Boucher 2001	2000	1.00	0.50	0.58	0.31	*0.98*	**1.16**	*3.74*	**1.088**	*4.00*	**1.15**

Col. 1: Net anthropogenic forcing ΔQ_1 to year shown, given by the listed source authority;
Col. 2: Estimated radiative imbalance ΔN_1 (based on Smith et al., 2015);
Col. 3: Observed warming ΔT_1 from 1850 to the end year shown (Morice et al., 2012);
Col. 4: Period reference sensitivity $\Delta R_1 = \Delta Q_1 P_2$;
Col. 5: *Current* period equilibrium sensitivity $\Delta E_1 = \Delta R_1 (A_M)_2 \quad | \quad (A_M)_2 = 3.14$;
Col. 6: Revised period equilibrium sensitivity ΔE_1 $(= \Delta T_1 \, \Delta Q_1 / (\Delta Q_1 - \Delta N_1)$;
Col. 7: Current system-gain factor $a_2 \approx \Delta E_1 / \Delta R_1$ (i.e., *Col. 5* / Col. 4);
Col. 8: Revised system-gain function $A_2 \approx E_2 / R_2$;
Col. 9: Current Charney sensitivity $\Delta E_2 = \Delta Q_2 P_2 a_2$ (i.e., $\Delta Q_2 P_2$ x *Col. 7*); and
Col. 10: Revised Charney sensitivity $\Delta E_2 = \Delta Q_2 P_2 A_2$ (i.e., $\Delta Q_2 P_2$ x **Col. 8**), to nearest 0.05 K.

The 2 σ uncertainties in Charney sensitivity ΔE_2 were computed via a Monte Carlo process. First, the uncertainty in each variable on which ΔE_2 depends was specified. Normal distributions were used, with the parameters derived from each variable's midrange estimate and stated 2 σ uncertainty bounds, under the conservative assumption that the uncertainties in all such variables were mutually independent. Charney sensitivity ΔE_2 depends on ΔR_2 (which in turn depends on $\Delta Q_2, P_2$), and on f_1 (which in turn depends on $R_0, \Delta R_0$). Thus, ΔQ_2 was taken as $3.447 \text{ W m}^{-2} \pm 5\%$; P_2 as $0.305\ [0.295, 0.315] \text{ K W}^{-1} \text{ m}^2$; R_0 as $254.7 \text{ K} \pm 5\%$; ΔR_0 was $10.3\ [8.9, 11.7]$ K; and E_1 as 287.55 ± 0.05 K in 1850 (Morice et al., 2012).

The overall uncertainty in ΔE_2 was obtained by simulating $n = 300{,}000$ draws from each variable on which ΔE_2 depends. Additional draws for the feedback fraction f_1 were made and inserted into Eq. (1) to obtain ΔE_2. For comparison, the uncertainty in ΔE_2 was likewise obtained using the CMIP5 ensemble's implicit feedback fraction $(f_M)_2$. The 2 σ bounds were estimated directly from the samples (Fig. 11).

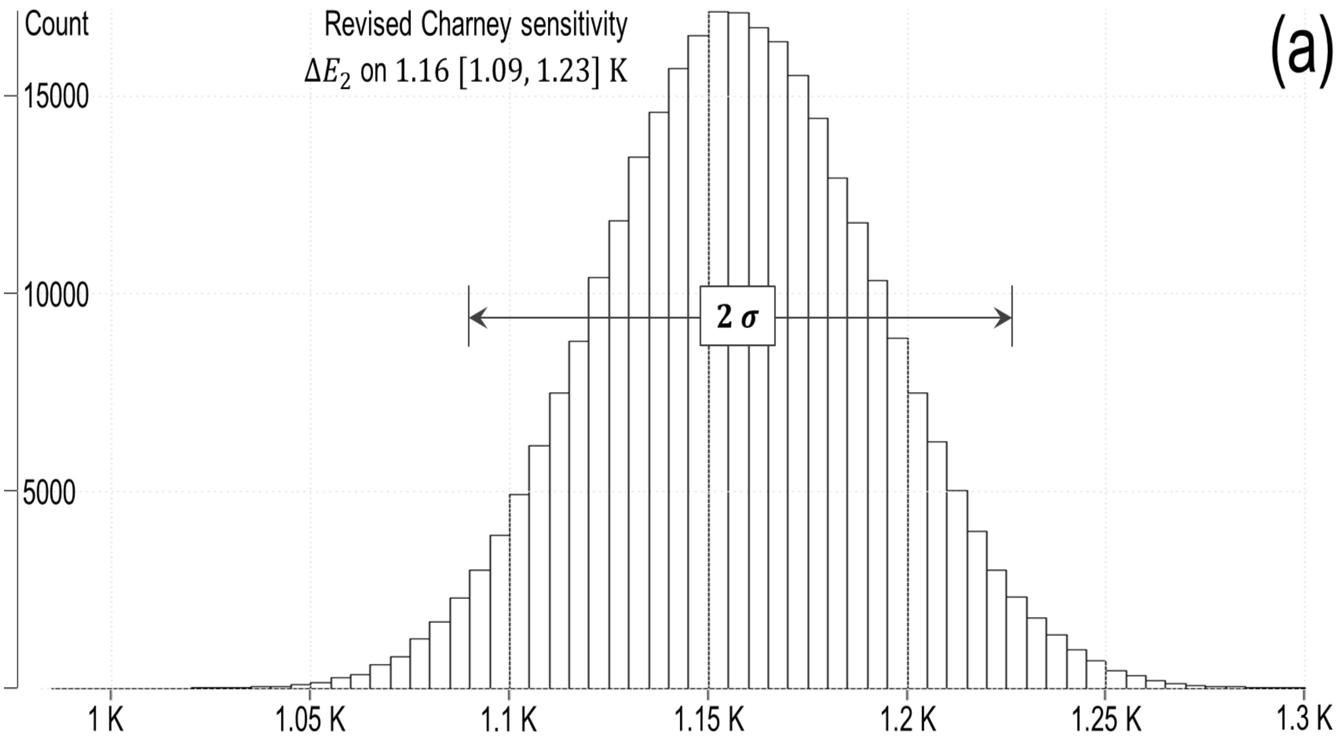

FIG. 11 (a) Monte Carlo distribution of Charney sensitivities ΔE_2 revised after correction of the error in defining temperature feedback identified herein. Bin widths are 0.005 K.

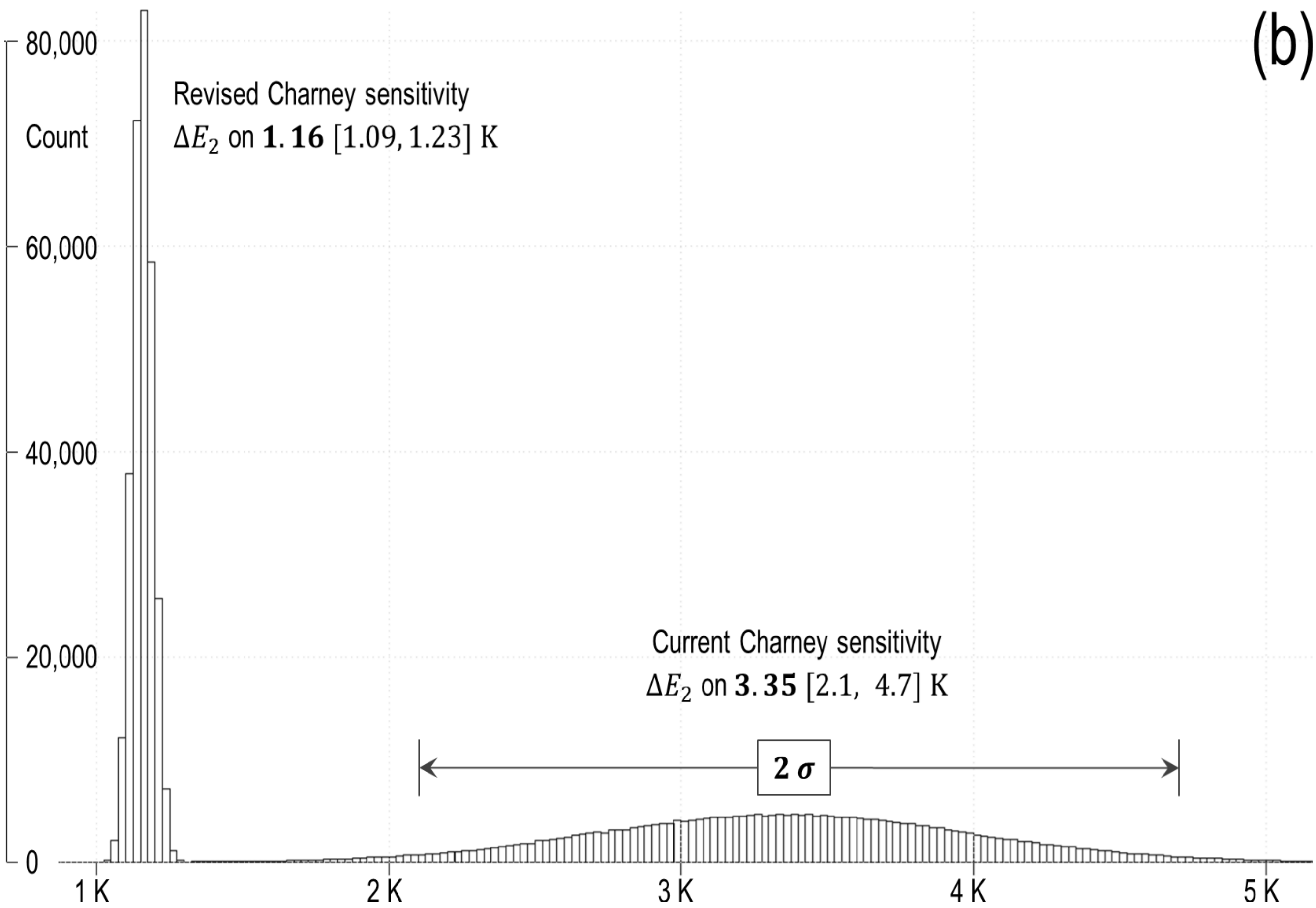

FIG. 11 (b) Scaled comparison of Monte Carlo distributions for revised (left) against current (right) Charney sensitivities ΔE_2. Here, bin widths are 0.025 K.

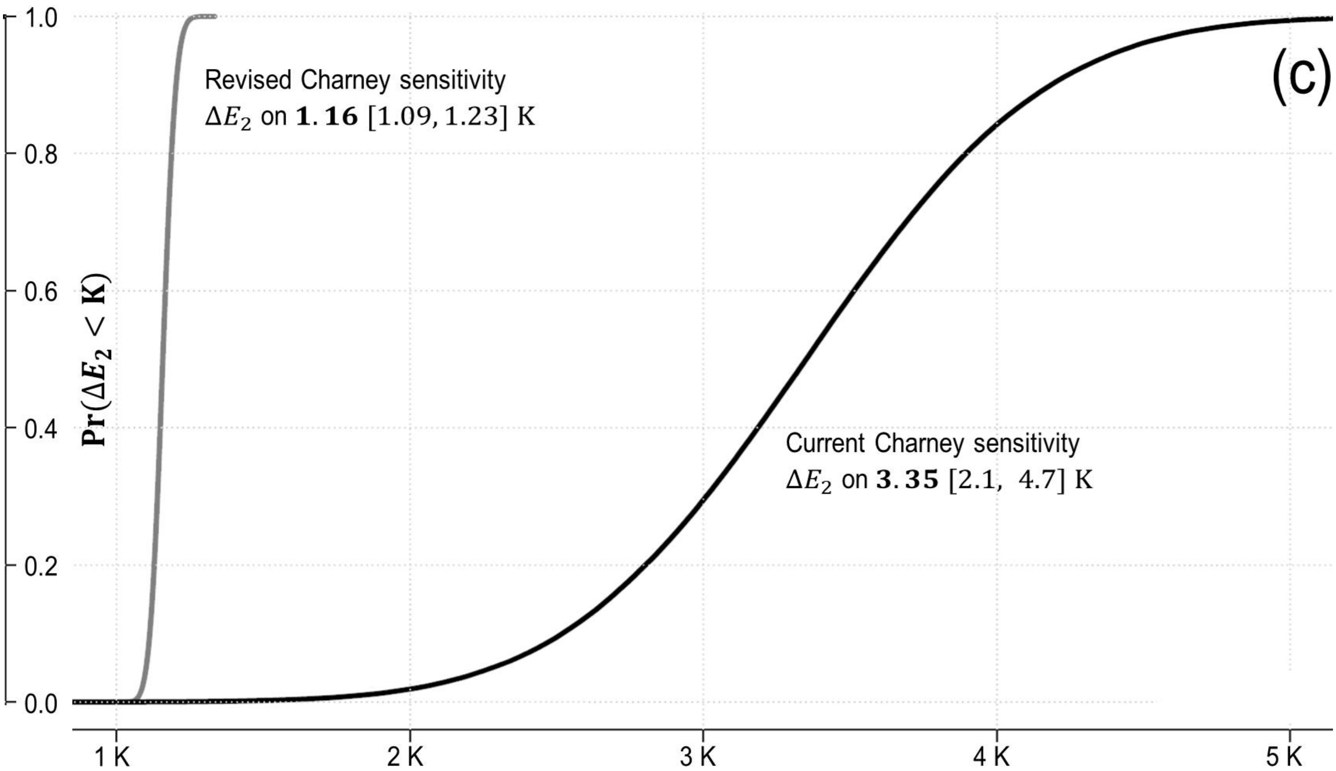

FIG. 11 (c) Cumulative distributions of probability that Charney sensitivity is less than a given value in K for revised (gray) against current (black) Charney sensitivities ΔE_2.

There is no consensus on the anthropogenic fraction of warming since 1850 (Legates et al., 2015). Here it is assumed to be 100%. If it is $< 100\%$, equilibrium sensitivity ΔE_2 may be less than shown.

9. Discussion and conclusion

Climatology erroneously defines temperature feedback as responding solely to anthropogenic perturbation, when feedback also responds to emission temperature and to natural perturbations thereof. This error of definition has engendered many consequential errors. For instance, climatology defines the partial system-gain factor a as the secant-slope of the curve of the equilibrium-sensitivity function $E(R)$, when it is demonstrated here that, regardless of the shape of $E(R)$, the true system-gain factor A_t at any time t is simply the ratio of E_t to R_t. Furthermore, climatology implicitly assumes not only that equilibrium sensitivity is time-independent but also that feedback response (especially the response to the water-vapor feedback) is exponential, when in truth feedback response, even to the water-vapor feedback, is close to linear, so that equilibrium sensitivity is at worst weakly time-dependent.

Though the shape of the equilibrium-temperature response function $E(R)$ is unknown, in models 1-4 it is here assumed to be an exponential-growth curve. The reason is that any net-growth curve other than an exponential-growth curve will, at some point t, demonstrate a still larger and still less plausible difference between the feedback fraction in response to emission temperature and the feedback fraction in response to a subsequent natural or anthropogenic perturbation of emission temperature.

Not the least reason why near-linearity in today's temperature-feedback regime is to be expected is that reference sensitivity R_1 in 1850 is 92% of equilibrium sensitivity E_1. Consequently, climatology's method implies that the feedback response to the NCGHGs exceeds the feedback response to emission temperature by one or even two orders of magnitude. Any such elevated feedback-fraction ratio is impossible, particularly given that the reference-sensitivity interval of interest, $[R_0, R_3]$, is narrow.

Correction of these errors by the use of the absolute system-gain factor A_t in Eq. (1) gives rise to a well constrained expectation of 1.15 [1.10, 1.25] K Charney sensitivity. That interval is in line with observation (Fig. 1) and with current estimates of net anthropogenic forcing in the industrial era. By contrast, the more volatile equation (Eq. 2) currently universally adopted in climatology delivers an interval excessive both in its magnitude and in its long-unconstrained breadth.

Once the large feedback response to the entire reference signal R_2 rather than solely to the anthropogenic perturbation ΔR_1 is correctly accounted for, the absolute system-gain factor A_3 and thus Charney sensitivity ΔE_2 prove to be well below current estimates. Since the CMIP3/5 ensemble implicitly assumes a system-gain factor $(A_M)_3 = 3.2$ (based on data in Andrews et al. 2012), deploying that assumption in Eq. (2) suggests models over-predict global warming to a greater extent than authorities such as Millar et al. (2017) have suggested. The revised midrange Charney sensitivity, at 1.15 K, is one-third of the implicit 3.35 K midrange CMIP5 estimate (also derived from Andrews, *op. cit.*). Contrary to suggestions (e.g. by Frame & Stone, 2013) that predictions in IPCC (1990) were accurate, the data underlying both Fig. 1 and the empirical campaign (Table 9) suggest that current GCMs very greatly overstate the system-gain factor and consequently Charney sensitivity.

Many explanations for the discrepancy between prediction and observation (Fig. 1) exist. Grose et al. (2017) suggest "global warming holes" (regions warming more slowly than average). Rahmstorf et al. (2012) find that removing short-term cooling influences like La Niña aligns the predictions in IPCC (1990) with observation. Occam's Razor, however, suggests that the substantial credibility gap between predicted and observed warming rates persists because GCMs' outputs reflect the definitional error identified here. Consequently, substantial overestimates of global warming have been made throughout the 120 years since Arrhenius (1896, table VII) first estimated Charney sensitivity as $\sim$5.5 K.

Though IPCC has hitherto assigned ever-greater "certainties" to the notion that recent warming was chiefly anthropogenic, that notion – poorly supported in the journals (Legates et al. 2015) – arises from the error of physics described here. Insofar as GCMs' equilibrium sensitivities reflect results obtained exclusively via Eq. (2), those sensitivities are greatly overstated. Climatology's attempt to apply a concept from experimental science in an observational-science setting has not succeeded. It is now advisable greatly to reduce what Hourdin et al. (2017) call the "anticipated acceptable range" of equilibrium sensitivities that models have hitherto been tuned to deliver. These results imply that, even without mitigation, there will be little net harm from the slow, small global warming that is to come.

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REFERENCES

Andrews T, Gregory JM, Webb MJ, Taylor KE (2012) Forcing, feedbacks and climate sensitivity in CMIP5 coupled atmosphere-ocean climate models. Geophys Res Lett 39:L09712, https://doi.org/10.1029/2012GL051607

Andronova NG, Schlesinger ME (2001) Objective estimation of the probability density function for climate sensitivity. J Geophys Res (Atmos) 106:22605-22611

Armour KC (2017) Energy budget constraints on climate sensitivity in light of inconstant climate feedbacks. Nature Climate Change 7:331-335, https://doi.org/ 10.1038/NCLIMATE3278

Arrhenius S (1896) On the influence of carbonic acid in the air upon the temperature of the ground. Philos Mag & J Sci 5(41):237-276

Bates JR (2016) Estimating climate sensitivity using two-zone energy-balance models. Earth & Space Sci 3:207-225, https://doi.org/10.1002/2015EA000154

Black HS (1934) Stabilized feedback amplifiers. Bell System Tech. J. (January) 1-18

Bode HW (1945) Network analysis and feedback amplifier design. Van Nostrand Reinhold, New York

Bony S, Colman R, Kattsov VM, Allan RP, Bretherton CS, Dufresne J-L, Hall A, Hallegatte S, Holland MM, Ingram W, Randall DA, Soden BJ, Tselioudis G, Webb MJ (2006) How well do we understand and evaluate climate change feedback processes? J Clim 19:3445-3482, https://doi.org/10.1175/JCLI3819.1

Boucher O, Haywood J (2001) On summing the components of radiative forcing of climate change. Clim Dyn 18:293-302, https://doi.org/10.1007/s003820100185

Cess RD, Zhang MH, Potter GL, Barker HW, Colman RA, Dazlich DA, Del Genio AD, Esch M, Fraser JR, Galin V, Gates WL, Hack JJ, Ingram WJ, Kiehl JT, Lacis AA, Le Treut L, Li Z-X, Liang X-Z, Mahfouf J-F, McAvaney BJ, Meleshko VP, Morcrette J-J, Randall DA, Roeckner E, Royer J-F, Sokolov AP, Sporyshev PV, Taylor KE, Wang W-C, Wetherald RT (1993) Uncertainties in carbon dioxide radiative forcing in atmospheric general-circulation models. Science 262(5137):1252-1255

Charney JG, Arakawa A, Baker DJ, Bolin B, Dickinson RE, Goody RM, Leith CE, Stommel HM, Wunsch CI (1979) Carbon Dioxide and Climate: A Scientific Assessment. Report of an Ad-Hoc Study Group on Carbon Dioxide and Climate, Climate Research Board, Assembly of Mathematical and Physical Sciences, National Research Council, Woods Hole, Massachusetts

Christy JR, Herman BM, Pielke R Sr., Klotzbach P, McNider RT, Hnilo JJ, Spencer RW, Chase T, Douglass DH (2010) What do observational datasets say about modeled tropospheric temperature trends since 1979? Remote Sens., 2, 2148-2169, doi:10.3390/rs2092148

Connolly R, Connolly R, Soon W, Legates DR, Cionco RG, Velasco Herrera VM (2019) Northern Hemisphere snow-cover trends (1967-2018): A comparison between climate models and observations. Geosci. 9:135, doi:10.3390/geosciences9030135

Dewitte S, Nevens S (2016) The total solar irradiance climate data record. https://doi.org/10.3847/0004-637X/830/1/25

Douglass DH, Christy JR, Pearson BD, Singer SF (2008) A comparison of tropical temperature trends with model predictions, Int. J. Climatol. 27:169301701

Fiedler S, Stevens B, Mauritsen T (2017) On the sensitivity of anthropogenic aerosol forcing to model-internal variability and parameterizing a Twomey effect, J. Adv. Model. Earth Syst. 9: doi:10.1002/2017MS000932

Friedrich T, Timmermann A, Tigchelaar M, Ganopolski A (2016) Nonlinear climate sensitivity and its implications for future greenhouse warming. Sci Adv 2(11), https://doi.org/10.1126/sciadv.1501923

Forest CE, Stone PH, Sokolov AP (2006) Estimated PDFs of climate system properties including natural and anthropogenic forcings. Geophys Res Lett 33:L01705

Frame DJ, Stone DA (2013) Assessment of the first consensus prediction on climate change. Nature Clim Change 3:357-359, https://doi.org/10.1038/nclimate1763

Fu Q, Manabe S, Johanson CM (2011) On the warming in the tropical upper troposphere: models versus observations, Geophys. Res. Lett., 38:L15704, doi:10.1029/2011GL048101

Gregory JM, Ingram WJ, Palmer MA, Jones GS, Stott PA, Thorpe RB, Lowe JA, Johns TC, Williams KD (2004) A new method for diagnosing radiative forcing and climate sensitivity. Geophys Res Lett 31:L03205, https://doi.org/10.1029/2003GL018747

Grose MR, Risbey JS, Whetton PH (2017) Tracking regional temperature projections from the early 1990s in light of variations in regional warming, including 'warming holes'. Climatic Change 140:307-322, https://doi.org/10.1007/s10584-016-1840-9

Hansen J, Johnson D, Lacis A, Lebedeff S, Lee P, Rind D, Russell G (1981) Climate impact of increasing atmospheric carbon dioxide. Science 213(4511):957-966

Hansen J, Lacis A, Rind D, Russell G, Stone P, Fung I, Ruedy R, Lerner J (1984) Climate sensitivity: Analysis of feedback mechanisms. In: Climate Processes and Climate Sensitivity, AGU

Geophysical Monograph 29, Maurice Ewing Vol. 5, Hansen J, Takahashi T (eds.), American Geophysical Union, 130-163

Harde H (2017) Radiation system-gain calculations and assessment of global warming by CO_2. Int J Atmos Sci, https://doi.org/10.1155/2017/9251034

Haywood J, Schulz M (2007) Causes of the reduction in uncertainty in the anthropogenic radiative forcing of climate between IPCC (2001) and IPCC (2007). Geophys Res Lett 34(10):L20701, https://doi.org/10.1029/2007GL030749

Hourdin F, Mauritsen T, Gettelman A, Golaz J-C, Balaji V, Duan Q, Folini D, Ji D, Klocke D, Qian Y, Rauser F, Rio C, Tomassini L, Watanabe M, Williamson D (2017) The art and science of climate model tuning. Bull. Amer. Met Soc. 98:3:589-603, doi:10.1175/BAMS-D-15-00135.1

IPCC (1990) Climate change – The IPCC Assessment (1990): Report prepared for the Intergovernmental Panel on Climate change by Working Group I. Houghton JT, Jenkins GJ, Ephraums JJ (eds.), Cambridge University Press, Cambridge

IPCC (2001) Climate Change 2001: The scientific basis. Contribution of Working Group I to the Third Assessment Report of the Intergovernmental Panel on Climate Change. Houghton JT, Ding Y, Griggs DJ, Noguer M, van der Lindin PJ, Dai X, Maskell K, Johnson CA (eds.), Cambridge University Press, Cambridge

IPCC (2007) Climate Change 2007: The physical science basis. Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change. Solomon S, Qin D, Manning M, Marquis M, Averyt K, Tignor MMB, LeRoy Miller H Jr, Chen Z (eds.), Cambridge University Press, Cambridge

IPCC (2013) Climate change 2013: The physical science basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change. Stocker TF, Qin D, Plattner G-K, Tignor MMB, Allen SK, Boschung J, Nauels A, Xia Y, Bex V, Midgley PM (eds.), Cambridge University Press, Cambridge

Johansson DJA, O'Neill NC, Tebaldi C, Häggström O (2015) Equilibrium climate sensitivity in light of observations over the warming hiatus. Nat Clim Change 5:449-453

Kalnay E, Kanamitsu M, Kistler R, Collins W, Deaven D, Gandin L, Iredell M, Saha S, White G, Woollen J, Zhu Y, Leetmaa A, Reynolds R (1996) The NCEP/NCAR Reanalysis 40-year Project. Bull Amer Meteor Soc 77:437-471

Knutti R, Stocker TF, Joos F. Plattner GK (2002) Constraints on radiative forcing and future climate change from observations and climate model ensembles. Nature 416:719-723, https://doi.org/10.1038/416979a

Lacis AA, Schmidt GA, Rind D, Ruedy RA (2010) Atmospheric CO_2: principal control knob governing Earth's temperature. Science 330:356-359

Legates DR, Soon WW-H. Briggs W, Monckton of Brenchley CW (2015) Climate consensus and misinformation: a rejoinder to "Agnotology Scientific Consensus, and the Teaching and Learning of Climate Change". Sci Edu, https://doi.org/10.1007/s11191-013-9647-9

Lewis N, Curry J (2018) The impact of recent forcing and ocean heat uptake data on estimates of climate sensitivity. J Clim, https://doi.org/10.1175/JCLI-D-17-0667.1

Loeb GN, Wielicki BA, Dölling DR, Smith GL, Keyes DF, Kato S, Manalo-Smith N, Wong T (2009) Toward optimal closure of the Earth's top-of-atmosphere radiation budget. J Clim 22:748-766, https://doi.org/10.1175/2008JCLI2637.1

McKitrick R, McIntyre S, Herman C (2010) Panel and multivariate methods for tests of trend equivalence in climate data series. Atmos. Sci. Lett. 11(4), https://doi.org/10.1002/asl.290

Mekaoui S, Dewitte S, Conscience C, Chevalier A (2010) Total solar irradiance absolute level from DIRAD/SOVIM on the International Space Station, Adv Space Res 45:1393-1406

Merlis TM, Schneider T (2010) Atmospheric dynamics of Earth-like tidally-locked aquaplanets. J Adv Model Earth Syst 2: https://doi.org/10.3894/JAMES.2010.2.13

Millar RJ, Fuglestvedt JS, Friedlingstein P, Rogelj J, Grubb MJ, Matthews HD, Skeie RB, Forster PM, Frame DJ, Allen MR (2017) Emission budgets and pathways consistent with limiting warming to 1.5 C°. Nature Geosci 10:741-747, 10.1038/ngeo3031

Miller RL, Schmidt GA, Nazarenko LS, Tausnev N, Bauer SE, Del Genio AD, Kelley M, Lo KK, Ruedy R, Shindell DT, Aleinov I, Bauer M, Bleck R, Canuto V, Chen Y-H, Cheng Y, Clune TL, Faluvegi G, Hansen JE, Healy RJ, Kiang NY, Koch D, Lacis AA, LeGrande AN, Lerner J, Menon S, Oinas V, Garcia-Pando CP, Perlwitz JP, Puma MJ, Rind D, Romaniou A, Russell GL, Sato M, Sun S, Tsigaridis K, Unger N, Voulgarakis A, Yao M-S, Zhang J (2014) CMIP5 historical simulations (1850-2012) with GISS ModelE2. J Adv Model Earth Syst 6(2):441-477, https://doi.org/ 10.1002/2013MS000265

Monckton of Brenchley CW (2011) Global brightening and climate sensitivity. In: Proceedings of the 42nd Annual International Seminars on Nuclear War and Planetary Emergencies, World Federation of Scientists (A. Zichichi and R. Ragaini, Eds.), World Scientific, London.

Monckton of Brenchley CW, Soon WW-H, Legates DN, Briggs WM (2015a) Why models run hot: results from an irreducibly simple climate model. Sci Bull 60(1):122-135, https://doi.org/10.1007/s11434-014-0699-2

Monckton of Brenchley CW (2015b) The temperature feedback problem. Energy Envir 26(5):829-840

Morice CP, Kennedy JJ, Rayner N, Jones PD (2012) Quantifying uncertainties in global and regional temperature change using an ensemble of observational estimates: The HadCRUT4 dataset. J Geophys Res 117:D08101, http://www.metoffice.gov.uk/hadobs/hadcrut4/data/current/ time_series/HadCRUT.4.5.0.0.monthly_ns_avg.txt, accessed 20 December 2018

Murphy DM, Solomon S, Portmann RW, Rosenlof KH, Forster PM, Wong T (2009) An observationally based energy balance for the Earth since 1950. J Geophys Res 114: D17107, https://doi.org/10.1029/2009D012105

Myhre G, Highwood EJ, Shine KP, Stordal F (1998) New estimates of radiative forcing due to well-mixed greenhouse gases. Geophys Res Lett 25:2715-2718

Myhre G, Myhre CL, Forster PM, Shine KP (2017) Halfway to doubling of CO_2 radiative forcing. Nature Geosci 10:710-711, https://doi.org/10.1038/ngeo3036

Paltridge G, Arking A, Pook M (2009) Trends in middle-and upper-level tropospheric humidity from NCEP reanalysis data, Theor. Appl. Climatol. 98(3):351-359

Pierrehumbert RT (2011) Infrared radiation and planetary temperature. Physics Today 64(1):33-38, https://doi.org/10.1063/1.3541943

Pinker RT, Zhang B, Dutton EG (2005) Do satellites detect trends in surface solar radiation? Science 308(5723):850-854, https://doi.org/10.1126/science.1103159

Rahmstorf S, Foster G, Cazenave A (2012) Comparing climate projections to observations up to 2011. Envir Res Lett 7: https://doi.org/10.1088/1748-9326/7/4/044035

Ranamathan V, Cess RD, Harrison EF, Minnis P, Barkstrom BR, Ahmad E, Hartmann D (1989) Cloud-radiative forcing and climate: results from the Earth Radiation Budget Experiment. Science 243(4887):57-63, https://doi.org/10.1126/science.243.4887.57

Roe G (2009) Feedbacks, timescales, and seeing red. Ann Rev Earth Planet Sci 37:93-115

Rosenblum E, Eisenman I (2017) Sea ice trends in climate models only accurate in runs with biased global warming. J Clim 30: 6265-6278, https://doi.org/10.1175/JCLI-D-16-0455.1

Sato Y, Goto D, Michibata T, Suzuki K, Takemura T, Tomita H, Nakajima T (2018) Aerosol effects on cloud water amounts were successfully simulated by a global cloud-system resolving model. Nature communic. 9(1):985, https://doi.org/10.1038/s41467-018-03379-6DO

Santer BD, Thorne PW, Haimberger L, Taylor KE, Wigley TML, Lanzante JR, Solomon S, Free M et al (2008) Consistency of modelled and observed temperature trends in the tropical troposphere, Int. J. Climatol., doi:1002/joc.1756

Schlesinger ME (1985) Feedback analysis of results from energy balance and radiative-convective models. In: The potential climatic effects of increasing carbon dioxide. MacCracken MC, Luther FM (eds,), US Dept of Energy, Washington DC, 280-319

Schmidt GA, Ruedy RA, Miller RL, Lacis AA (2010) Attribution of the present-day total greenhouse effect. J Geophys Res (Atmos) 115:D20106, https://doi.org/ 10.1029/2010JD014287

Sherwood SC, Nishant N (2015) Atmospheric changes through 2012 as shown by iteratively homogenized radiosonde temperature and wind data (IUKv2). Environ. Res. Lett., http://dx.doi.org/10.1088/1748-9326/10/5/054007

Seifert A, Heus T, Pincus R, Stevens B (2015) Large-eddy simulation of the transient and near-equilibrium behavior of precipitating shallow convection. J. Advances in Modeling Earth Systems, 7:1918-1937: https://doi.org/10.1002/2015MS000489

Skeie RB, Berntsen TK, Myhre G, Tanaka K, Kvalevåg MM, Hoyle CR (2011) Anthropogenic radiative forcing time series from pre-industrial times until 2010. Atmos Chem Phys 11:11827-11857, https://doi.org/10.5194/acp-11-11827-2011

Smith DM, Allan RP, Coward AC, Eade R, Hyder P, Liu C, Loeb NG, Palmer MD, Roberts CD, Scaife AA (2015) Earth’s energy imbalance since 1960 in observations and CMIP5 models. Geophys Res Lett 42(4): https://doi.org/10.1002/2014GL062669

Soden BJ, Held IM (2006) An assessment of climate feedbacks in coupled ocean-atmosphere models. J Clim 19:3354-3360

Stevens B (2015) Rethinking the lower bound on aerosol radiative forcing. J. Clim. 28:4794-4819: https://doi.org/10.1175/JCLI-D-14-00656.1

Thorne PW, Lanzante JR, Peterson TC, Seidel DJ, Shine KP (2011) Tropospheric temperature trends: history of an ongoing controversy, WIRES: Climate Change 2:66-88

Vial J, Dufresne J-L, Bony S (2013) On the interpretation of inter-model spread in CMIP5 climate sensitivity estimates, Clim Dyn 41:3339-3362, https://doi.org/ 10.1007/s00382-013-1725-9

Wentz FJ, Ricciardulli L, Hilburn K, Mears C (2007) How much more rain will global warming bring? Science 317:233-235

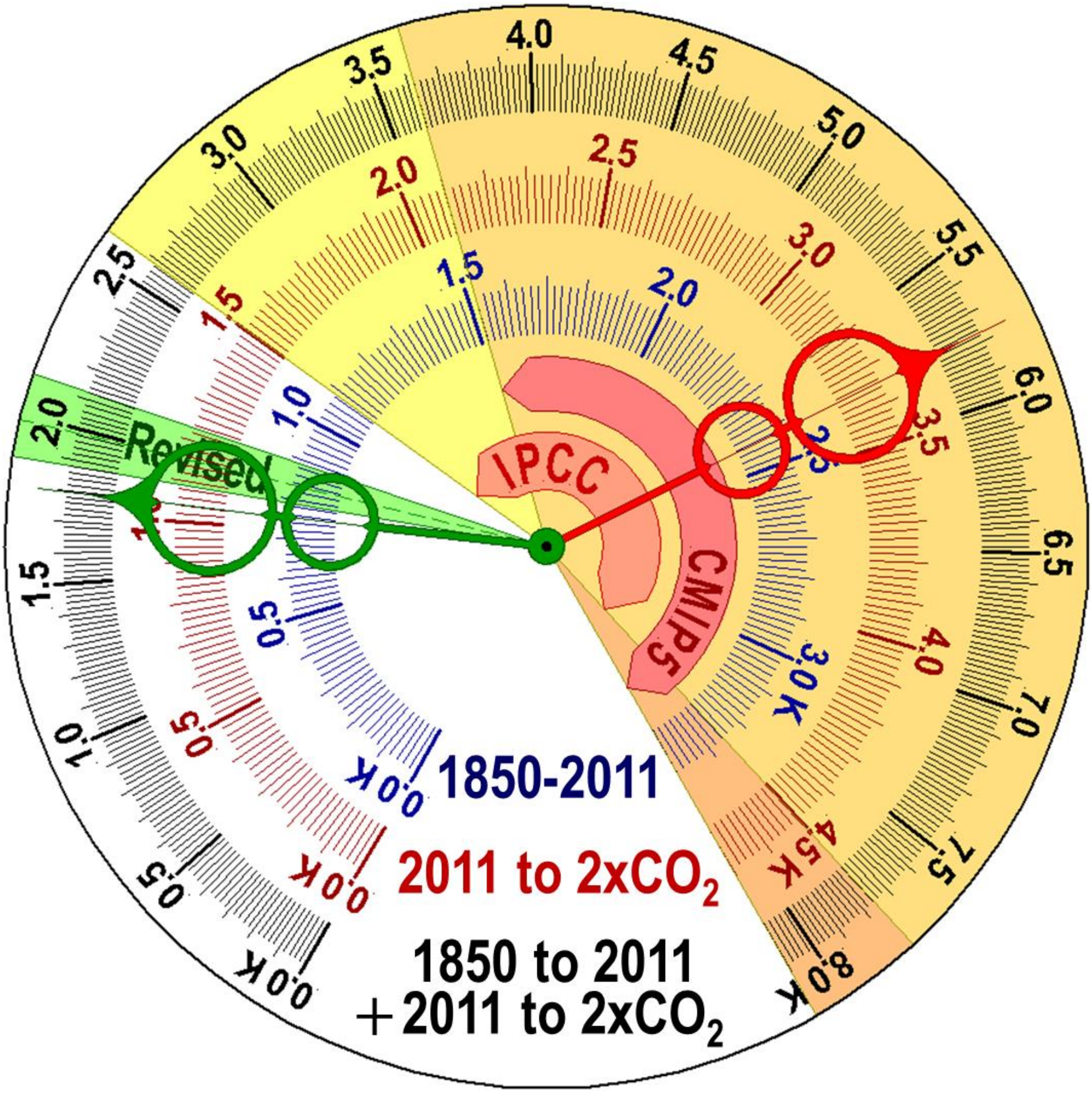

FIG. 1. Overlapping projections by IPCC (2013) and CMIP5 (Andrews et al. 2012) of global warming from 1850-2011 **(blue scale)**, in response to doubled CO_2 **(red scale)** and the sum of these two (**black scale**) greatly exceed warming equivalent to the 0.75 K observed from 1850-2011 (HadCRUT4: **green needle**). The 3.35 K CMIP5 mid-range Charney sensitivity **(red needle)** implies 2.4 K anthropogenic warming by 2011, about thrice observation. The revised warming interval derived herein **(pale green region)** is consistent with observed warming to 2011 (**green needle**).

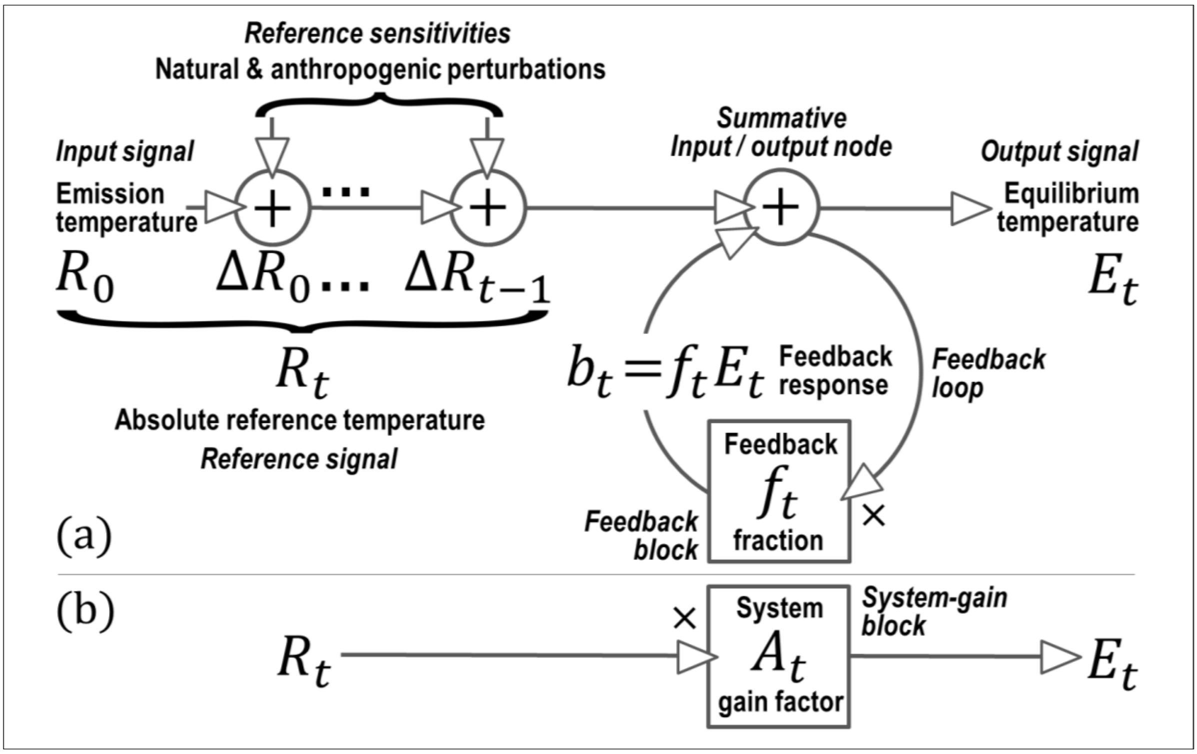

Fig. 2. The ***feedback loop*** (a) simplifies to (b), the schematic for the **system-gain factor** A_t at time t. The ***reference signal*** **(reference temperature** R_t), the sum of the ***input signal*** **(emission temperature** R_0), and all ***perturbations*** (**reference sensitivities** $\Delta R_0, \dots \Delta R_{t-1}$), is input via the ***summative input/output node*** to the feedback loop. The ***output signal*** **(equilibrium temperature** E_t), is the sum of R_t and the **feedback response** $b_t \coloneqq f_t E_t$ ($\coloneqq E_t - R_t$). Then $A_t \,(= E_t / R_t)$ is equal to the sum $\sum_{i=0}^{\infty} f_t^i = (1 - f_t)^{-1}$ of the infinite convergent geometric series $\{{f_t}^0 + {f_t}^1 + \cdots + {f_t}^\infty\}$ under the convergence criterion $|\, f_t \,| < 1$. The ***feedback block*** (a) and the ***system-gain block*** (b) must perforce act not only on the anthropogenic perturbation ΔR_{t-1} but on the entire reference signal R_t.

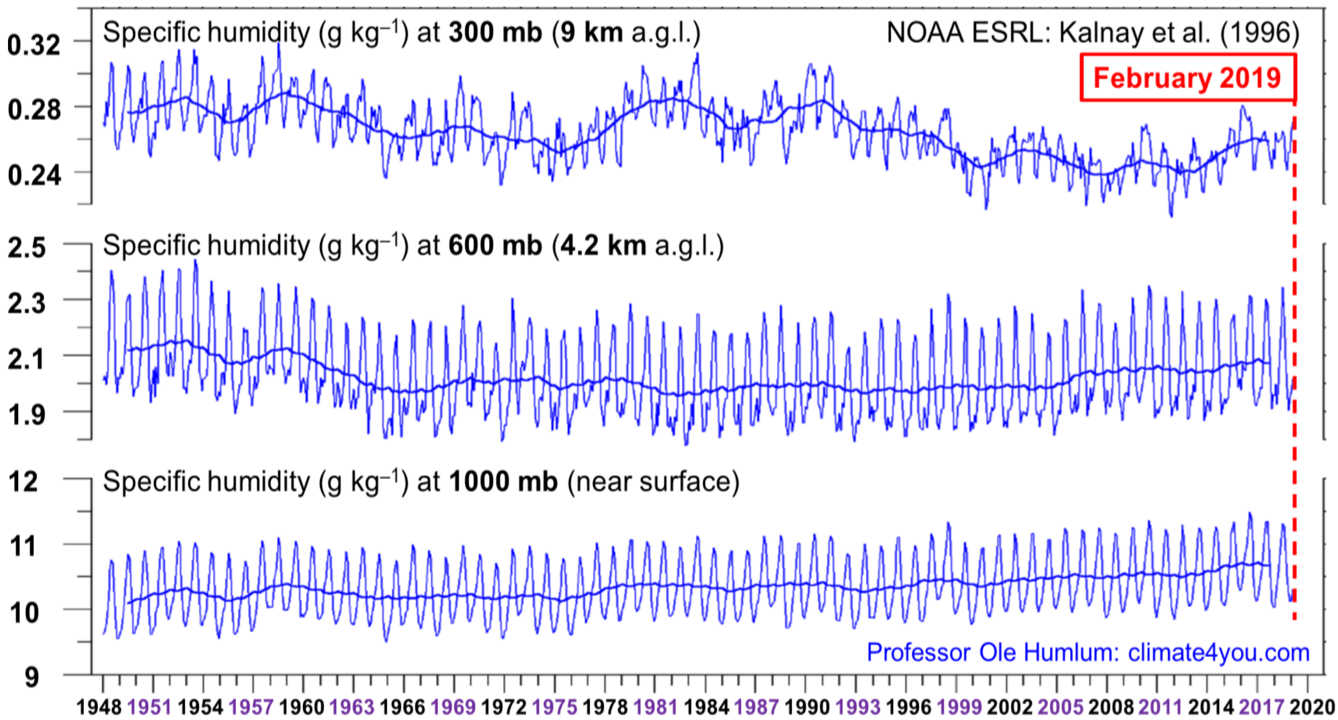

FIG. 3 Specific humidity (g kg^{-1}) at 300, 600 and 1000 mb

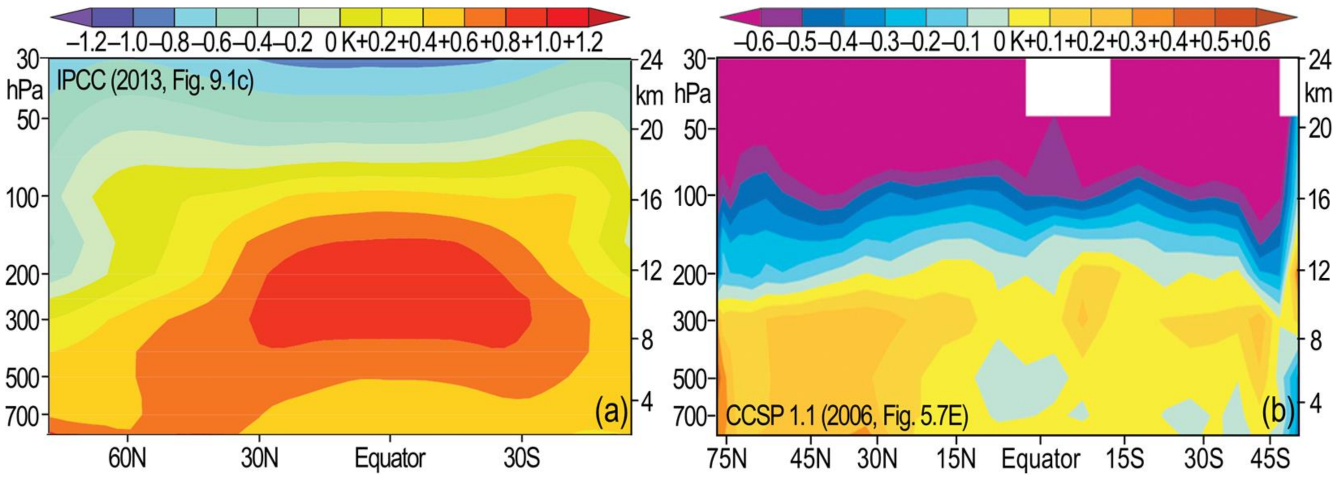

FIG. 4. | GCMs' projected "hot spot"[28] (a) is absent in observational data[11] (b). Temperature anomalies (in Kelvin) are color-coded.

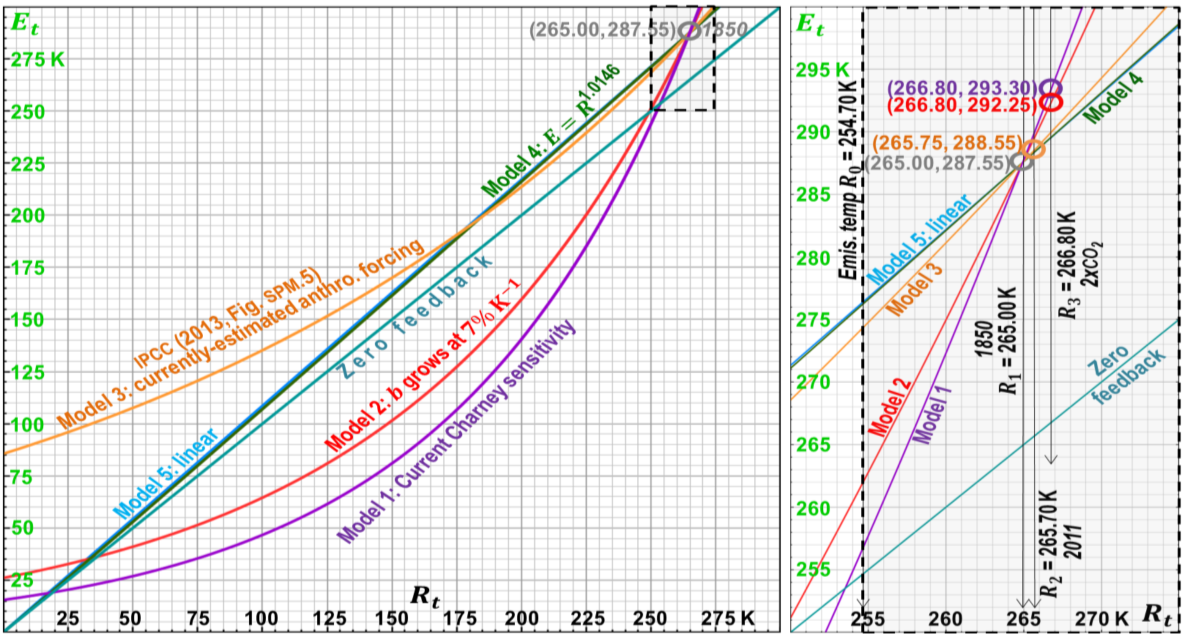

FIG. 5 Comparison of the five models of the evolution of $E(R)$ for R on $[250, 275]$ K. Models 1 (purple), 2 (red) and 3 (orange) are each generated from two points: the circled points in their colors and the common gray point $(265.00, 287.55)$ representing the position in 1850. In model 4 (green), the exponent $x = \ln(287.55)/\ln(265.70) = 1.0146$. Model 5 (pale blue) is the linear model. For comparison, the zero-feedback line $E = R$ is shown in turquoise. All five models appear linear across the interval R_t on $[250, 275]$ K (right panel). It is possible that the shape of the response function $E(R)$ is neither linear nor exponential. However, the fact that, owing to the dominance of emission temperature in the climate system, R_1 is more than 92% of E_1 strongly suggests that large departures from linearity in equilibrium-temperature response are not to be expected.

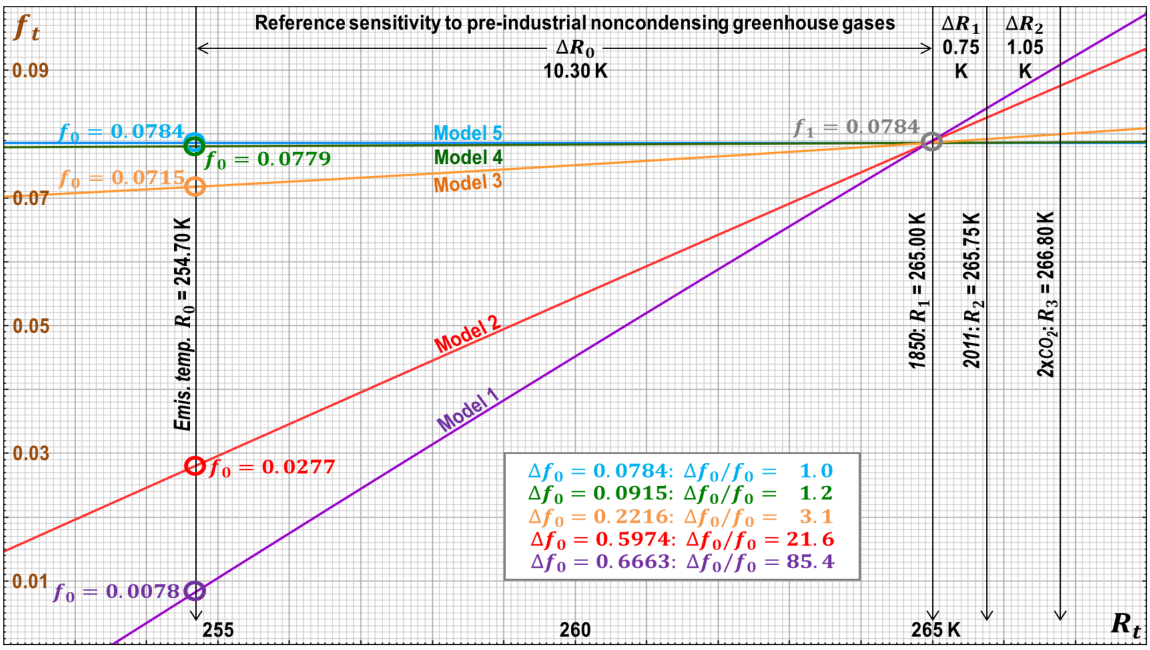

FIG. 6 Evolution of the feedback fraction f_t from emission temperature R_0 (= 254.7 K) to reference temperature R_1 (= 265.00 K) in 1850 and beyond.

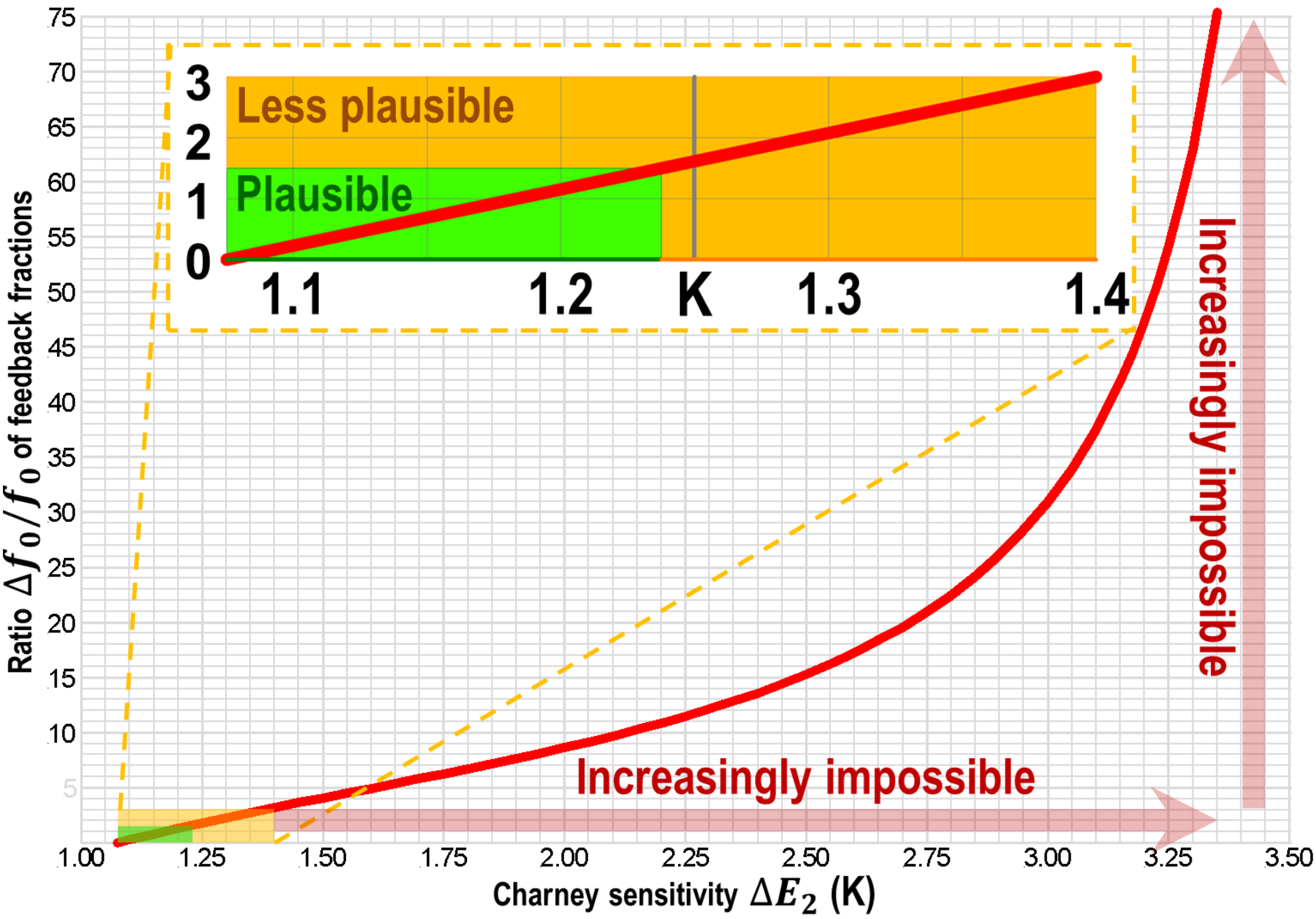

FIG. 7 The feedback-fraction ratio $\Delta f_0/f_0$, i.e., the ratio of the feedback fraction Δf_0 in response to reference sensitivity to the pre-industrial NCGHGs and the feedback fraction f_0 in response to emission temperature, for Charney sensitivity ΔE_2 on $[1.07, 3.35]$ K, for equilibrium temperature E_t an exponential-growth function $E(R)$ of reference temperature R_t. Green region: plausible sensitivities; orange region: implausible sensitivities. Beyond these, elevated feedback-fraction ratios and thus sensitivities are increasingly impossible.

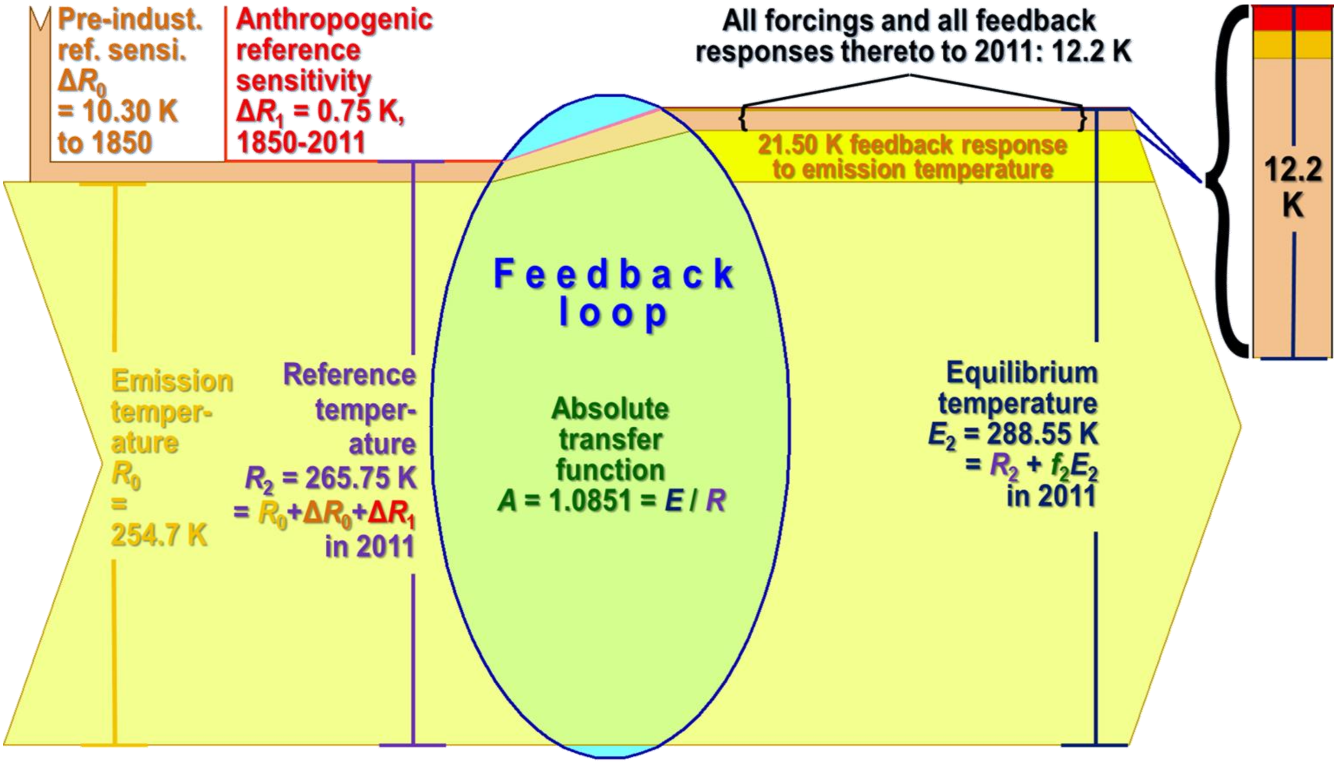

FIG. 8 Relative magnitudes of the contributions to reference temperature R_2 and to equilibrium temperature E_2 in 2011, assuming $E_t = g(R_t) = R_t^x$ (model 4). Dominance of emission temperature (pale yellow) and its feedback response (bright yellow) is visible.

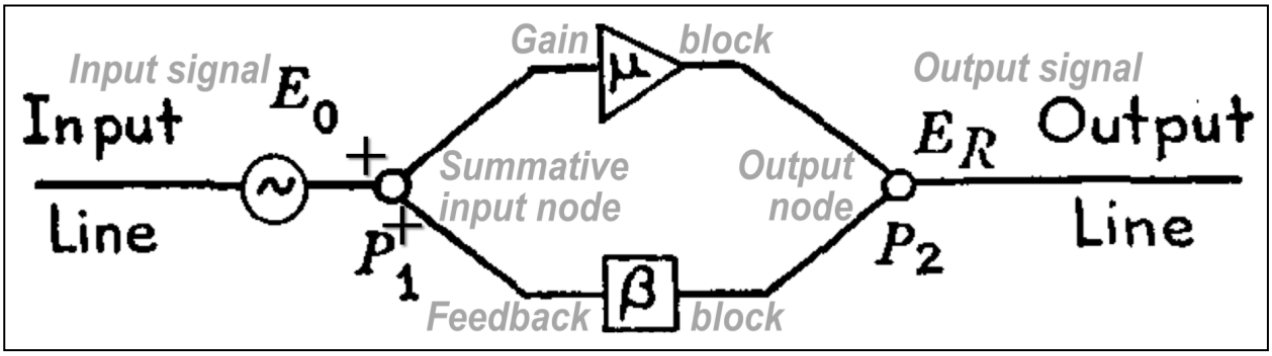

FIG. 9 A feedback amplifier with a μ gain block and a β feedback block (Bode 1945)

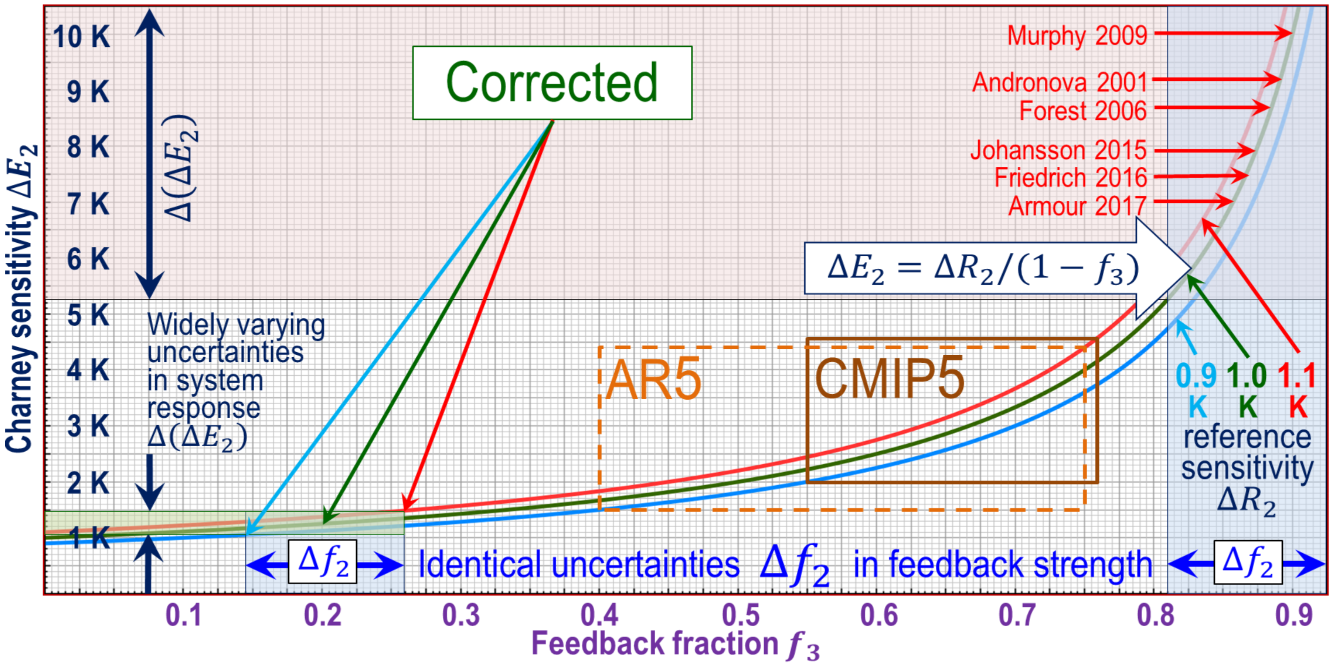

FIG. 10 The rectangular-hyperbolic response curves of Charney sensitivities ΔE_2 against feedback factors f_3 for reference sensitivity ΔR_2 on 1.0 K $\pm$ 10%. Identical uncertainties Δf_2 in f_3 generate broader uncertainty intervals $\Delta(\Delta E_2)$ in system response ΔE_2 as $f_3 \to 1$, since $\Delta(\Delta E_2)$ depends greatly on f_3 (Roe 2009, fig. 6). High-end predictions of ΔE_2 from six sources, the CMIP5 GCMs' interval [2.1, 4.7] K and the 2 σ interval 1.15 [1.10, 1.25] K from Eq. (1) are shown. Varying f_3 visibly makes very much more difference to ΔE_2 than varying ΔR_2, particularly where $f_3 \to 1$.

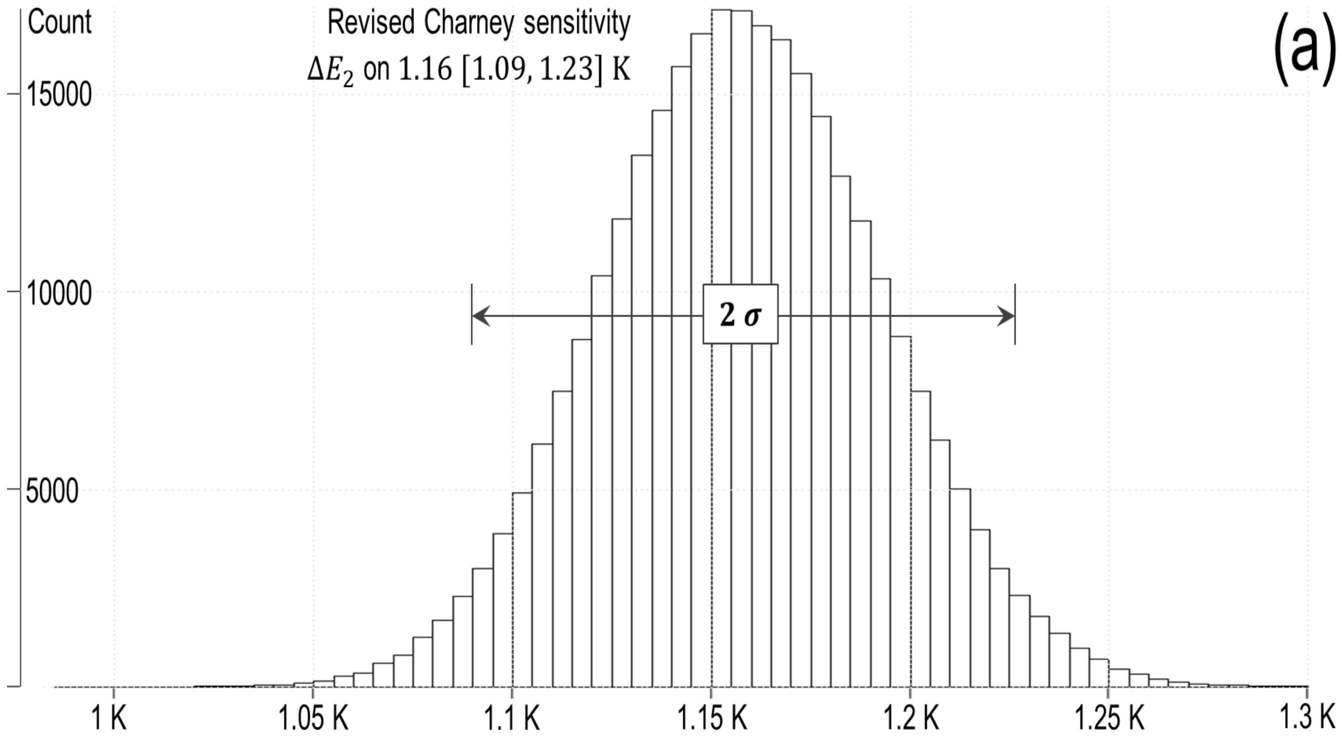

FIG. 11 (a) Monte Carlo distribution of Charney sensitivities ΔE_2 revised after correction of the error in defining temperature feedback identified herein. Bin widths are 0.005 K.

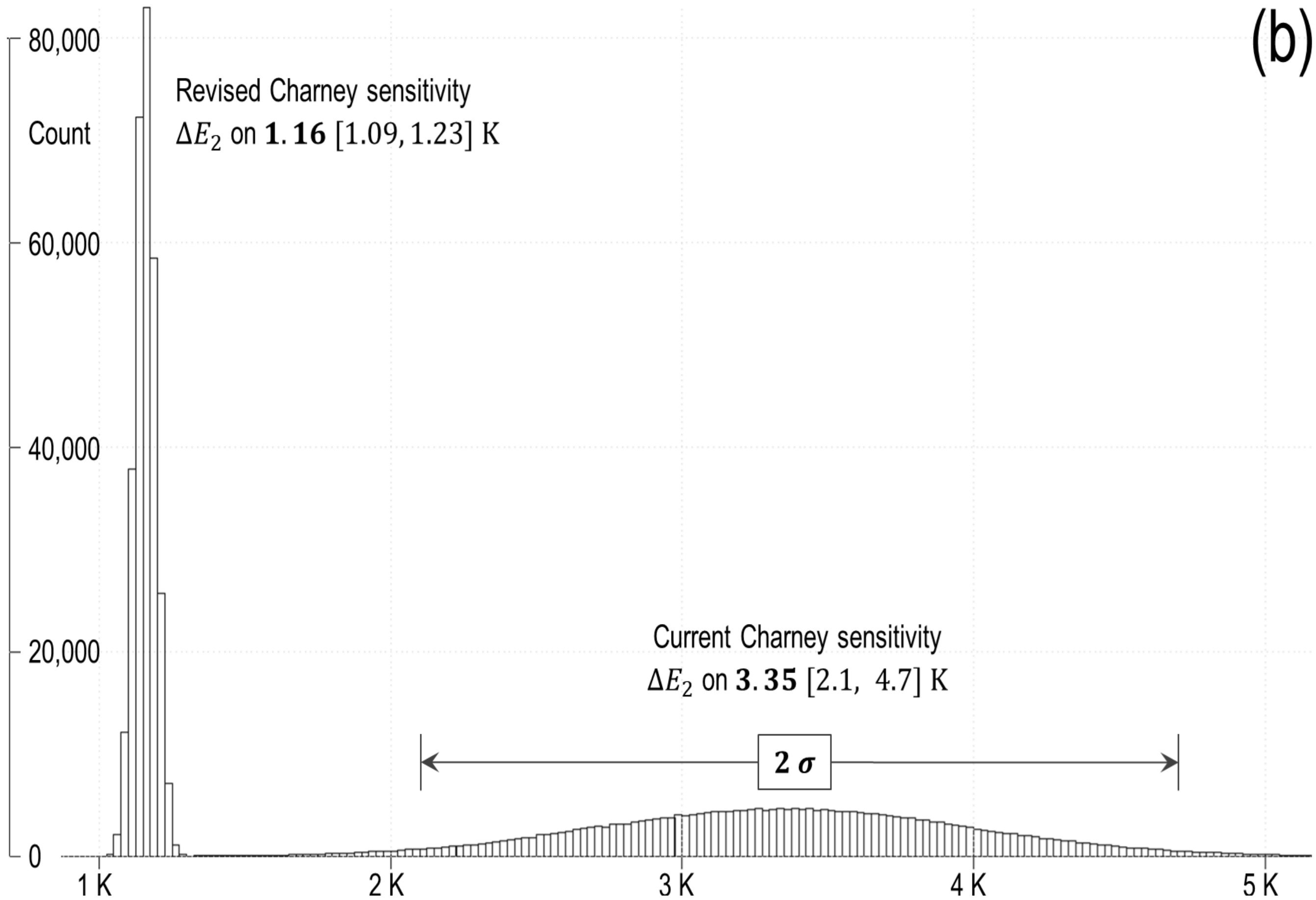

FIG. 11 (b) Scaled comparison of Monte Carlo distributions for revised (left) against current (right) Charney sensitivities ΔE_2. Here, bin widths are 0.025 K.

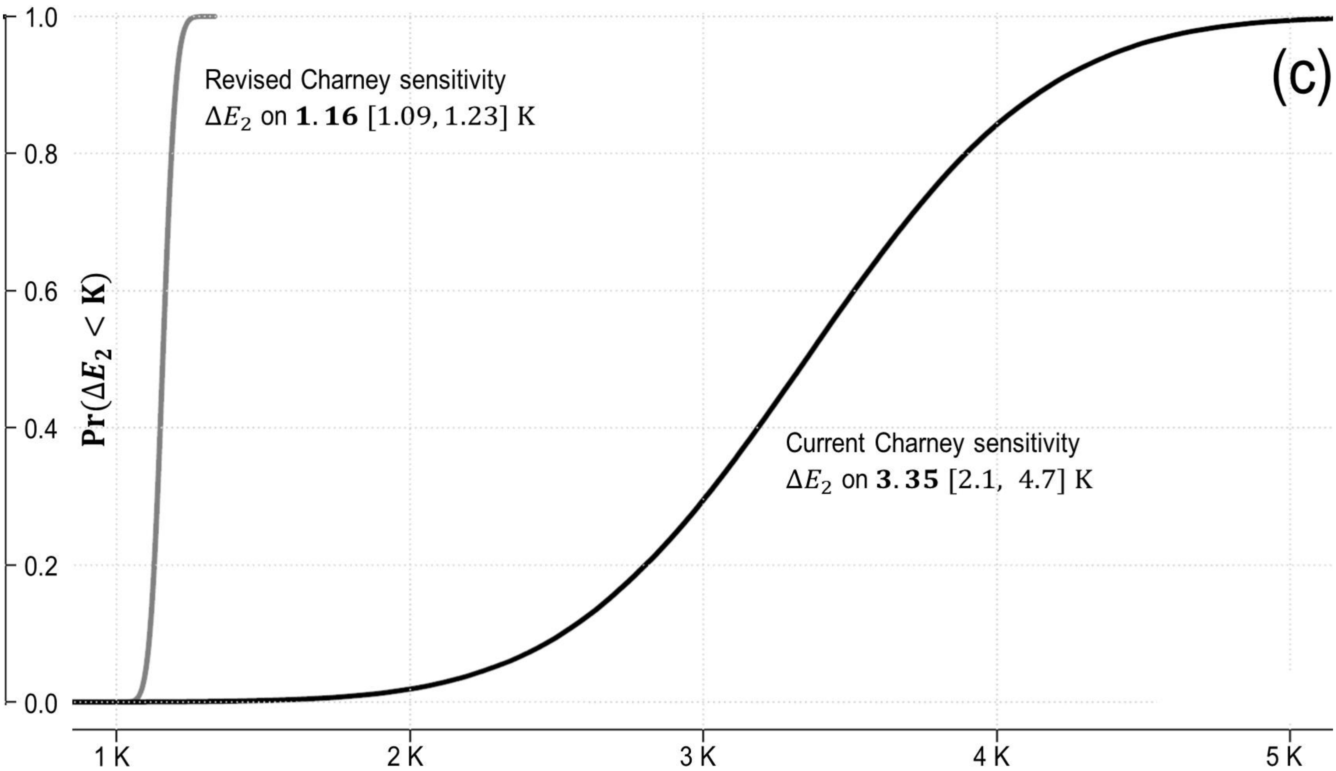

FIG. 11 (c) Cumulative distributions of probability that Charney sensitivity is less than a given value in K for revised (gray) against current (black) Charney sensitivities ΔE_2.

TABLES

TABLE 1 Current feedbacks based on IPCC (2013, p. 818, table 9.5 and p. 128, Fig. 1.2)

Temperature feedback	Lower bound	**Mid-range**	Upper bound	Timescale
Water vapor feedback $(\lambda_1)_2$	$+1.3$ W m^{-2} K^{-1}	**$+1.6$ W m^{-2} K^{-1}**	$+1.9$ W m^{-2} K^{-1}	Hours
Lapse rate feedback $(\lambda_2)_2$	-1.0 W m^{-2} K^{-1}	**-0.6 W m^{-2} K^{-1}**	-0.2 W m^{-2} K^{-1}	Hours
Cloud feedback $(\lambda_3)_2$	-0.4 W m^{-2} K^{-1}	**$+0.3$ W m^{-2} K^{-1}**	$+1.1$ W m^{-2} K^{-1}	Days
Surface albedo feedback $(\lambda_4)_2$	$+0.2$ W m^{-2} K^{-1}	**$+0.3$ W m^{-2} K^{-1}**	$+0.4$ W m^{-2} K^{-1}	Years
IPCC feedback sum $\lambda_2 = \sum_{i=1}^{4}(\lambda_i)_2$	$+0.1$ W m^{-2} K^{-1}	**$+1.6$ W m^{-2} K^{-1}**	$+3.2$ W m^{-2} K^{-1}	Years
IPCC feedback fraction $f_2 = \lambda_2 P_2$	[+0.0]	**$+0.5$**	[+1.0]	
IPCC system-gain factor $a_2 = 1/(1 - f_2)$	[1.0]	**2.0**	[Undefined]	
IPCC implicit Charney sensi. $E_3 = \Delta R_2 a_2$	[1.0]	**2.6**	[∞]	

TABLE 2 Evolution of midrange reference temperature R_t (to the nearest 0.05 K).

Emission temperature	Pre-industrial	In 1850	1850-2011	In 2011	2011-2xCO_2	At 2xCO_2
R_0	ΔR_0	R_1	ΔR_1	R_2	ΔR_2	R_3
254.70 K	10.30 K	265.00 K	0.75 K	265.75 K	1.05 K	266.80 K

TABLE 3 Results from model 1

Emiss. temp.		Pre-industrial		In 1850		1850-2011		In 2011		2011 to 2xCO_2		At 2xCO_2	
R_0	254.70 K	ΔR_0	10.30 K	R_1	265.00 K	ΔR_1	0.75 K	R_2	265.75 K	ΔR_2	1.05 K	R_3	266.80 K
E_0	256.70 K	ΔE_0	30.85 K	E_1	287.55 K	ΔE_1	2.40 K	E_2	289.95 K	ΔE_2	3.40 K	E_3	293.30 K
b_0	2.00 K	Δb_0	20.55 K	b_1	22.55 K	Δb_1	1.65 K	b_2	24.20 K	Δb_2	2.35 K	b_3	26.50 K
f_0	**0.0078**	**Δf_0**	**0.6663**	f_1	0.0784	Δf_1	0.6858	f_2	0.0834	Δf_2	0.6889	f_3	0.0904
A_0	1.0078	ΔA_0	2.9966	A_1	1.0851	ΔA_1	3.1831	A_2	1.0910	ΔA_2	3.2149	A_3	1.0994
s_0	2.8297	(a_0)	(2.9966)	s_1	3.1700	(a_1)	(3.1831)	s_2	3.1963	(a_2)	(3.2149)	s_3	3.2335

TABLE 4 Results from model 2

Emiss. temp.		Pre-industrial		In 1850		1850-2011		In 2011		2011 to 2xCO_2		At 2xCO_2	
R_0	254.70 K	ΔR_0	10.30 K	R_1	265.00 K	ΔR_1	0.75 K	R_2	265.75 K	ΔR_2	1.05 K	R_3	266.80 K
E_0	261.95 K	ΔE_0	25.60 K	E_1	287.55 K	ΔE_1	1.95 K	E_2	289.50 K	ΔE_2	2.75 K	E_3	292.25 K
b_0	7.25 K	Δb_0	15.30 K	b_1	22.55 K	Δb_1	1.20 K	b_2	23.75 K	Δb_2	1.70 K	b_3	25.50 K
f_0	**0.0277**	**Δf_0**	**0.5974**	f_1	0.0784	Δf_1	0.6169	f_2	0.0821	Δf_2	0.6200	f_3	0.0872
A_0	1.0285	ΔA_0	2.4841	A_1	1.0851	ΔA_1	2.6105	A_2	1.0894	ΔA_2	2.6318	A_3	1.0955
s_0	2.3701	(a_0)	(2.4841)	s_1	2.6016	(a_1)	(2.6105)	s_2	2.6194	(a_2)	(2.6318)	s_3	2.6444

Table 5 Results from model 3

Emiss. temp.		Pre-industrial		In 1850		1850-2011		In 2011		2011 to 2xCO_2		At 2xCO_2	
R_0	254.70 K	ΔR_0	10.30 K	R_1	265.00 K	ΔR_1	0.75 K	R_2	265.75 K	ΔR_2	1.05 K	R_3	266.80 K
E_0	274.30 K	ΔE_0	13.25 K	E_1	287.55 K	ΔE_1	1.00 K	E_2	288.55 K	ΔE_2	1.40 K	E_3	289.95 K
b_0	19.60 K	Δb_0	2.95 K	b_1	22.55 K	Δb_1	0.25 K	b_2	22.80 K	Δb_2	0.35 K	b_3	23.15 K
f_0	**0.0715**	**Δf_0**	**0.2216**	f_1	0.0784	Δf_1	0.2410	f_2	0.0790	Δf_2	0.2441	f_3	0.0798
A_0	1.0770	ΔA_0	1.2847	A_1	1.0851	ΔA_1	1.3175	A_2	1.0858	ΔA_2	1.3229	A_3	1.0867
s_0	1.2547	(a_0)	(1.2847)	s_1	1.3152	(a_1)	(1.3175)	s_2	1.3197	(a_2)	(1.3229)	s_3	1.3261

Table 6 Results from model 4

Emiss. temp.		Pre-industrial		In 1850		1850-2011		In 2011		2011 to 2xCO_2		At 2xCO_2	
R_0	254.70 K	ΔR_0	10.30 K	R_1	265.00 K	ΔR_1	0.75 K	R_2	265.75 K	ΔR_2	1.05 K	R_3	266.80 K
E_0	276.20 K	ΔE_0	11.35 K	E_1	287.55 K	ΔE_1	0.85 K	E_2	288.40 K	ΔE_2	1.15 K	E_3	289.55 K
b_0	21.50 K	Δb_0	1.05 K	b_1	22.55 K	Δb_1	0.10 K	b_2	22.65 K	Δb_2	0.10 K	b_3	22.75 K
f_0	**0.0779**	**Δf_0**	**0.0915**	f_1	0.0784	Δf_1	0.0918	f_2	0.0785	Δf_2	0.0918	f_3	0.0785
A_0	1.0845	ΔA_0	1.1007	A_1	1.0851	ΔA_1	1.1010	A_2	1.0852	ΔA_2	1.1011	A_3	1.0852
s_0	1.1004	(a_0)	(1.1007)	s_1	1.1010	(a_1)	(1.1010)	s_2	1.1010	(a_2)	(1.1011)	s_3	1.1011

Table 7 Results from model 5

Emiss. temp.		Pre-industrial		In 1850		1850-2011		In 2011		2011 to 2xCO_2		At 2xCO_2	
R_0	254.70 K	ΔR_0	10.30 K	R_1	265.00 K	ΔR_1	0.75 K	R_2	265.75 K	ΔR_2	1.05 K	R_3	266.80 K
E_0	276.35 K	ΔE_0	11.20 K	E_1	287.55 K	ΔE_1	0.80 K	E_2	288.35 K	ΔE_2	1.15 K	E_3	289.50 K
b_0	21.70 K	Δb_0	0.90 K	b_1	22.55 K	Δb_1	0.05 K	b_2	22.60 K	Δb_2	0.10 K	b_3	22.70 K
f_0	**0.0784**	**Δf_0**	**0.0784**	f_1	0.0784	Δf_1	0.0784	f_2	0.0784	Δf_2	0.0784	f_3	0.0784
A_0	1.0851	ΔA_0	1.0851	A_1	1.0851	ΔA_1	1.0851	A_2	1.0851	ΔA_2	1.0851	A_3	1.0851
s_0	1.0851	(a_0)	(1.0851)	s_1	1.0851	(a_1)	(1.0851)	s_2	1.0851	(a_2)	(1.0851)	s_3	1.0851

Table 8 Relationship between elevated ratios $\Delta f_0/f_0$ and elevated Charney sensitivities ΔE_2

	Model	b_0	Δb_0	f_0	Δf_0	$\Delta f_0/f_0$	A_3	ΔE_2
CMIP5 current ΔE_2:	1	2.00 K	20.55 K	**0.0078**	**0.6663**	**85**	1.0994	3.40 K
b_t + 1.07% K^{-1}:	2	7.25 K	15.30 K	**0.0277**	**0.5974**	**22**	1.0955	2.75 K
IPCC anth. forcing:	3	19.60 K	2.95 K	**0.0715**	**0.2216**	**3.1**	1.0867	1.40 K
$R = 0 \Rightarrow b = 0$:	4	21.50 K	1.05 K	**0.0779**	**0.0915**	**1.2**	1.0852	1.15 K
Linear:	5	21.70 K	0.90 K	**0.0784**	**0.0784**	**1.0**	1.0851	1.15 K

TABLE 9 System-gain factors A_2 and Charney sensitivities ΔE_2 in the industrial era to 2011

		ΔQ_1	ΔN_1	ΔT_1	ΔR_1	ΔE_1	$\boldsymbol{\Delta E_1}$	a_2	$\boldsymbol{A_2}$	ΔE_2	$\boldsymbol{\Delta E_2}$
		– W m^{-2} –		K				Unitless		K	
Data source	Year	1	2	3	4	*5*	**6**	*7*	**8**	*9*	**10**
Miller 2014	2012	2.95	0.60	0.76	0.91	*2.88*	**0.95**	*1.04*	**1.085**	*1.11*	**1.15**
Myhre 2017	2016	3.10	0.60	0.84	0.96	*3.03*	**1.04**	*1.08*	**1.085**	*1.16*	**1.15**
IPCC AR5 2013	1980	1.25	0.40	0.38	0.39	*1.22*	**0.56**	*1.44*	**1.086**	*1.54*	**1.15**
Knutti 2002	2001	1.90	0.50	0.62	0.59	*1.86*	**0.84**	*1.43*	**1.086**	*1.53*	**1.15**
IPCC AR5 2013	1950	0.57	0.20	0.26	0.18	*0.56*	**0.40**	*2.27*	**1.086**	*2.42*	**1.15**
IPCC AR5 2013	**2011**	**2.29**	**0.60**	**0.76**	**0.71**	***2.24***	**1.03**	***1.45***	**1.086**	***1.55***	**1.15**
Haywood 2007	2006	1.93	0.50	0.68	0.60	*1.88*	**0.92**	*1.53*	**1.086**	*1.64*	**1.15**
IPCC AR4 2007	2005	1.60	0.50	0.67	0.50	*1.56*	**0.97**	*1.96*	**1.087**	*2.10*	**1.15**
Skeie 2011	2010	1.40	0.60	0.74	0.43	*1.37*	**1.30**	*2.98*	**1.088**	*3.19*	**1.15**
Boucher 2001	2000	1.00	0.50	0.58	0.31	*0.98*	**1.16**	*3.74*	**1.088**	*4.00*	**1.15**

Col. 1: Net anthropogenic forcing ΔQ_1 to year shown, given by the listed source authority;

Col. 2: Estimated radiative imbalance ΔN_1 (based on Smith et al., 2015);

Col. 3: Observed warming ΔT_1 from 1850 to the end year shown (Morice et al., 2012);

Col. 4: Period reference sensitivity $\Delta R_1 = \Delta Q_1 P_2$;

Col. 5: *Current* period equilibrium sensitivity $\Delta E_1 = \Delta R_1 (A_M)_2 \quad | \quad (A_M)_2 = 3.14$;

Col. 6: Revised period equilibrium sensitivity ΔE_1 $(= \Delta T_1\, \Delta Q_1 / (\Delta Q_1 - \Delta N_1)$;

Col. 7: Current system-gain factor $a_2 \approx \Delta E_1 / \Delta R_1$ (i.e., *Col. 5* / Col. 4);

Col. 8: Revised system-gain function $A_2 \approx E_2 / R_2$;

Col. 9: Current Charney sensitivity $\Delta E_2 = \Delta Q_2 P_2 a_2$ (i.e., $\Delta Q_2 P_2$ x *Col. 7*); and

Col. 10: Revised Charney sensitivity $\Delta E_2 = \Delta Q_2 P_2 A_2$ (i.e., $\Delta Q_2 P_2$ x **Col. 8**), to nearest 0.05 K.

FIGURE CAPTIONS

FIG. 1. Overlapping projections by IPCC (2013) and CMIP5 (Andrews et al. 2012) of global warming from 1850-2011 **(blue scale)**, in response to doubled CO_2 **(red scale)** and the sum of these two (**black scale**) greatly exceed warming equivalent to the 0.75 K observed from 1850-2011 (HadCRUT4: **green needle**). The 3.35 K CMIP5 mid-range Charney sensitivity **(red needle)** implies 2.4 K anthropogenic warming by 2011, about thrice observation. The revised warming interval derived herein **(pale green region)** is consistent with observed warming to 2011 (**green needle**).

FIG. 2. The ***feedback loop*** (a) simplifies to (b), the schematic for the **system-gain factor** A_t at time t. The ***reference signal*** **(reference temperature** R_t), the sum of the ***input signal*** **(emission temperature** R_0), and all ***perturbations*** (**reference sensitivities** $\Delta R_0, \ldots \Delta R_{t-1}$), is input via the ***summative input/output node*** to the feedback loop. The ***output signal*** **(equilibrium temperature** E_t), is the sum of R_t and the **feedback response** $b_t \coloneqq f_t E_t$ ($\coloneqq E_t - R_t$). Then $A_t (= E_t / R_t)$ is equal to the sum $\sum_{i=0}^{\infty} f_t^i = (1 - f_t)^{-1}$ of the infinite convergent geometric series $\{f_t^{\,0} + f_t^{\,1} + \cdots + f_t^{\,\infty}\}$ under the convergence criterion $|\, f_t \,| < 1$. The ***feedback block*** (a) and the ***system-gain block*** (b) must perforce act not only on the anthropogenic perturbation ΔR_{t-1} but on the entire reference signal R_t.

FIG. 3 Specific humidity (g kg^{-1}) at 300, 600 and 1000 mb

FIG. 4 GCMs' projected "hot spot"[28] (a) is absent in observational data[11] (b). Temperature anomalies (in Kelvin) are color-coded.

FIG. 5 Comparison of the five models of the evolution of $E(R)$ for R on $[250, 275]$ K. Models 1 (purple), 2 (red) and 3 (orange) are each generated from two points: the circled points in their colors and the common gray point $(265.00, 287.55)$ representing the position in 1850. In model 4 (green), the exponent $x = \ln(287.55)/\ln(265.70) = 1.0146$. Model 5 (pale blue) is the linear model. For comparison, the zero-feedback line $E = R$ is shown in turquoise. All five models appear linear across the interval R_t on $[250, 275]$ K (right panel). It is possible that the shape of the response function $E(R)$ is neither linear nor exponential. However, the fact that, owing to the dominance of emission temperature in the climate system, R_1 is more than 92% of E_1 strongly suggests that large departures from linearity in equilibrium-temperature response are not to be expected.

FIG. 6 Evolution of the feedback fraction f_t from emission temperature R_0 (= 254.7 K) to reference temperature R_1 (= 265.00 K) in 1850 and beyond.

FIG. 7 The feedback-fraction ratio $\Delta f_0/f_0$, i.e., the ratio of the feedback fraction Δf_0 in response to reference sensitivity to the pre-industrial NCGHGs and the feedback fraction f_0 in response to emission temperature, for Charney sensitivity ΔE_2 on $[1.07, 3.35]$ K, for equilibrium temperature E_t an exponential-growth function $E(R)$ of reference temperature R_t. Green region: plausible sensitivities; orange region: implausible sensitivities. Beyond these, elevated feedback-fraction ratios and thus sensitivities are increasingly impossible.

FIG. 8 Relative magnitudes of the contributions to reference temperature R_2 and to equilibrium temperature E_2 in 2011, assuming $E_t = g(R_t) = R_t^x$ (model 4). Dominance of emission temperature (pale yellow) and its feedback response (bright yellow) is visible.

FIG. 9 A feedback amplifier with a μ gain block and a β feedback block (Bode 1945)

FIG. 10 The rectangular-hyperbolic response curves of Charney sensitivities ΔE_2 against feedback factors f_3 for reference sensitivity ΔR_2 on 1.0 K $\pm$ 10%. Identical uncertainties Δf_2 in f_3 generate broader uncertainty intervals $\Delta(\Delta E_2)$ in system response ΔE_2 as $f_3 \to 1$, since $\Delta(\Delta E_2)$ depends greatly on f_3 (Roe 2009, fig. 6). High-end predictions of ΔE_2 from six sources, the CMIP5 GCMs' interval [2.1, 4.7] K and the $2\,\sigma$ interval $1.15\ [1.10, 1.25]$ K from Eq. (1) are shown. Varying f_3 visibly makes very much more difference to ΔE_2 than varying ΔR_2, particularly where $f_3 \to 1$.

FIG. 11 (a) Monte Carlo distribution of Charney sensitivities ΔE_2 revised after correction of the error in defining temperature feedback identified herein. Bin widths are 0.005 K.

FIG. 11 (b) Scaled comparison of Monte Carlo distributions for revised (left) against current (right) Charney sensitivities ΔE_2. Here, bin widths are 0.025 K.

FIG. 11 (c) Cumulative distributions of probability that Charney sensitivity is less than a given value in K for revised (gray) against current (black) Charney sensitivities ΔE_2.